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Limitations on the Attainable Intensity of High Power Lasers

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It is shown that even a single e^-e^+ pair created by a superstrong laser field in vacuum would cause development of an avalanchelike QED cascade which rapidly depletes the incoming laser pulse. This confirms Bohr's old conjecture that the electric field of the critical QED strength $E_S = m^2c^3/e\hbar$ could never be created.

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Sauter was the first to introduce the critical QED field $E_S = m^2c^3/e\hbar = 1.32 \times 10^{16}\ \mathrm{V/cm}$ in his remarkable papers [1], where he considered the so-called Klein paradox [2]. The content of the paradox is that, according to the Dirac theory, an electron can penetrate into a potential barrier which is greater than twice the rest energy of the electron. Sauter was also the first to interpret the Klein paradox as the creation of e^-e^+ pairs by an external field from vacuum.

According to Sommerfeld, Bohr suggested a postulate that the electric field of the strength E_S could never be created in principle; see Chap. IV Sect. 11 in [3]. However, further development of QED has demonstrated that no difficulties arise in the theory at $E \sim E_S$. The Klein paradox was successively resolved [1,4,5], the nonlinear correction to electromagnetic Lagrangian valid at arbitrary field strength was calculated [6,7], and many authors theoretically considered physics at field strength higher than the critical value. Thus the Bohr's conjecture was completely forgotten.

It was shown recently [8–10] that pairs can be created in vacuum by focused laser pulses with the peak field strength below the critical value. Simultaneous focusing of multiple colliding pulses may lower the threshold field strength for the process down to $10^{-2}E_S$ [11]. The corresponding optical laser intensity is of the order $5 \times 10^{25}\ \mathrm{W/cm^2}$. Projects (e.g., ELI, XFEL) to achieve such intensity in the near future are under way already [12–14], so that pair creation from vacuum by external electromagnetic fields, thought as a sort of gedanken experiment for years, may become accessible for experimental observation very soon.

Dependence of the average number of created pairs on field strength is determined by the exponential factor $\exp(-\pi E_S/E)$ and is very sharp. At $E \sim E_S$ the estimated number of produced pairs $N_{e^-e^+}$ is so huge that their total rest energy becomes comparable with the energy of the laser pulse itself [8–10]. Certainly, $N_{e^-e^+}$ at $E \sim E_S$ was overestimated in Refs. [8–10] because it was obtained under assumption that the laser field could be considered as a given classical background. However, that result indicated that the process of pair creation would lead to depletion of the laser field and the effect of backreaction should be taken into account. Moreover, this estimate could serve as an argument in favor of Bohr's postulate on the unattainability of pair-creating electromagnetic fields with $E = E_S$.

However, there exists another, and even more effective, mechanism for depletion of a pair-creating laser pulse. The point is that the created electron and positron can be very quickly accelerated by the laser field to relativistic energies and emit hard photons, which produce in turn new e^-e^+ pairs [15]. These effects have been already observed in the famous E144 SLAC experiment [16], but yet just as single events, because the energy of electrons and hard photons, as well as the laser intensity were not high enough. At high laser intensities interaction of the created electron and positron with the laser field can lead to production of multiple new particles and thus to the formation of an avalanchelike electromagnetic cascade [17–19]. Such cascade have many features in common with the cascades produced as the result of a high-energy particle interacting with dense matter. The latter were well studied both experimentally and theoretically, see, e.g., Ref. [20]. However, there exists an important distinctive feature of the laser-induced cascades, as compared with the air showers arising due to primary cosmic ray entering atmosphere. In our case the laser field plays not only the role of a target (similar to a nuclei in the case of air showers). It is responsible also for acceleration of slow particles [17], playing thus the role of a linac in the SLAC experiment. Thus, the laser-induced cascade in vacuum looks very much like electron avalanche which can occur due to impact ionization in dielectric-filled trench used for electrical isolation of semiconductor devices [21]. In this letter we will consider the mechanism of onset and development of an electromagnetic cascade initiated by a pair created in vacuum by short focused laser pulses. We will show that creation of even a single pair may result in complete destruction of the laser field.

Consider a primary pair created from vacuum by an ultrarelativistic ($a_0 = eE/m\omega c \gg 1$) but still subcritical ($E, H \ll E_S$) focused laser field. By virtue of the condition $a_0 \gg 1$ the formation length of any quantum process in such a field is much less than the wavelength λ, $l_f/\lambda \sim 1/a_0 \ll 1$. Hence, the field can be considered locally constant [22]. Besides, under this condition the created particles are characterized by a bare, rather than an effective mass which is a characteristic of averaged motion of a particle in a plane wave field, and hence can be introduced only if $l_f \gtrsim \lambda$.

The total probability of pair creation from vacuum is determined by local values of the field invariants

$$\mathcal{F} = (E^2 - H^2)/2E_S^2, \qquad \mathcal{G} = (\boldsymbol{E} \cdot \boldsymbol{H})/E_S^2.$$

The secondary processes of photon emission by a charged particle and pair creation by a photon depend in addition on the dynamical parameter χ,

$$\chi = \frac{e\hbar}{m^3 c^4}\sqrt{-(F_{\mu\nu}p^\nu)^2},$$

where p^ν is 4-momentum either of electron (positron), or photon. The parameter χ for an electron is exactly its proper acceleration in the field measured in Compton units $mc^3/\hbar$. In the lab frame it is proportional to $\sqrt{f_\parallel^2 + \gamma_e^2 f_\perp^2}$, where $\boldsymbol{f}_{\parallel,\perp}$ are the components of the Lorentz force, longitudinal and transverse to the electron motion, respectively, γ_e is the electron Lorentz factor.

If $\mathcal{F}$, $\mathcal{G} \ll \chi^2$, then the probability W of any process initiated by a particle can be approximated by $W(\mathcal{F}, \mathcal{G}, \chi) \approx W(0, 0, \chi)$, which is the probability of the same process in a plane wave field. Since the condition $l_f \ll \lambda$ is satisfied, W is the probability rate of the process in a constant crossed field [23].

The rates for both photon emission and pair creation by photon in such a field can be easily found in the literature, see, e.g., [22]. The total probability rates in the limiting case of large χ read

$$W_{e,\gamma} \sim \frac{\alpha m^2 c^4}{\hbar \epsilon_{e,\gamma}} \chi_{e,\gamma}^{2/3}, \tag{1}$$

where ϵ_e, ϵ_γ denote the energy of the initial particle. In the classical limit $\chi_\gamma \ll 1$ the rate of pair creation by a photon is exponentially suppressed by the factor $\exp(-8/3\chi_\gamma)$.

Particles of the pair created by the field from vacuum can be considered to be at rest initially $\chi_e(0) = E/E_S \ll 1$. The cascade can develop if the particles are able to emit hard photons with $\chi_\gamma \gtrsim 1$, i.e., if $\chi_e \gtrsim 1$. This means that the act of hard photon emission can take place if the field accelerates particles and the time of acceleration $t_{\rm acc}$ is small enough, $t_{\rm acc} \ll t_{\rm esc}$ at least, where $t_{\rm esc}$ is the time of stay of the particle in the laser pulse. For a laser pulse focused up to the diffraction limit the time $t_{\rm esc}$ may be estimated to be $t_{\rm esc} \sim \lambda/2c$.

The process of acceleration of a particle is not a local event. Hence, to describe it we cannot apply the "constant field approximation." To estimate $t_{\rm acc}$, we will use the model of a uniform purely electric field rotating with frequency ω. Such field can be realized practically in the antinodes of a circularly polarized monochromatic standing wave. The initial electron and positron will be accelerated by the field in opposite directions. We will consider below only one branch of the cascade initiated by the positron. Since no hard photons can be emitted at $t < t_{\rm acc}$, we will treat the motion of the particle classically. The equation of motion $\dot{\boldsymbol{p}}_e(t) = e\boldsymbol{E}(t)$ can be easily solved. The result is presented by solid line in Fig. 1. Parameter χ_e in this case is not conserved as, e.g., in a constant field but oscillates with period $2\pi/\omega$ and the amplitude of oscillations $\chi_{e\,\max} \approx 2(E/E_S)a_0$ can considerably exceed unity even for relatively moderate field intensities. Figure 1 contains also the results of numerical calculations for the cases of crossed orthogonally polarized plane waves and e-polarized tightly focused laser beam. It is clear that although the details of evolution of $\chi_e(t)$ are much more complicated in general case, the basic qualitative features, such as the period and the amplitude of oscillations are of the same order as in the case of a uniformly rotating electric field. Therefore, the subsequent estimation for

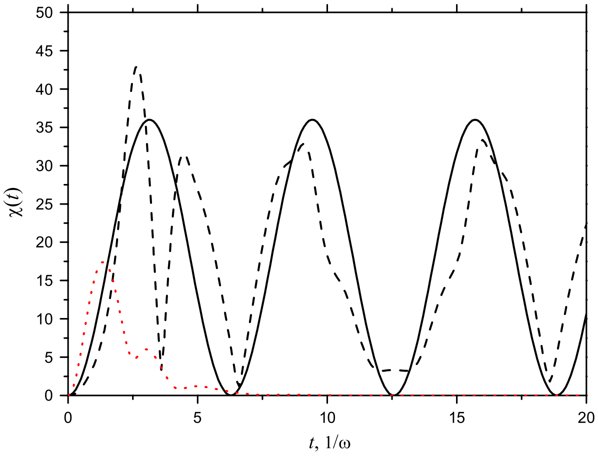

FIG. 1 (color online). Evolution of quantum dynamical parameter χ along the particle trajectory for $a_0 = 3 \times 10^3$, $\hbar\omega = 1$ eV in three cases: head-on collision of two elliptically polarized plane waves (solid line); collision at 90° of two linearly polarized plane waves with orthogonal linear polarizations (dashed line); single tightly focused e-polarized laser beam (dotted line).

$t_{\rm acc}$ is valid for any pairs creating laser field, at least qualitatively.

Consider parameter $\chi_e(t)$ at $t \ll \pi/\omega$. It is determined by the product of the positron Lorentz factor, in our case $\gamma_e \sim eEt/mc$, and the transverse component of the field. It can be easily observed that the angle between the field and particle momentum increases as $\theta_e = \omega t/2$. Thus, we estimate $E_\perp \sim E\omega t$ and come to

$$\chi_e(t) \sim \left(\frac{E}{E_S}\right)^2 \frac{mc^2\omega t^2}{\hbar}. \tag{2}$$

The crucial point here is that the rotating field not only accelerates the particle but also contorts its trajectory, so that the Lorentz force gains transverse component. As it is seen from the Eq. (2), $t_{\rm acc}$ can be estimated as

$$t_{\rm acc} \sim \frac{\hbar}{\alpha mc^2\mu}\sqrt{\frac{mc^2}{\hbar\omega}}, \tag{3}$$

where $\mu = E/E_*$ and $E_* = \alpha E_S \approx E_S/137$. For optical frequency, this time period remains just a small fraction of the rotation period provided that $I > 10^{24}$ W/cm^2.

The positron radiation lifetime (mean free path/c) t_e with respect to photon emission can be estimated by $t_e \sim W_e^{-1}$. Thus, at the moment of photon emission we have

$$\epsilon_e \sim eE\frac{c}{W_e}, \qquad \chi_e \sim \left(\frac{E}{E_S}\right)^2 \frac{mc^2\omega}{\hbar W_e^2}. \tag{4}$$

After substitution of (4) into Eq. (1), we find that $\chi_e \sim \mu^{3/2}$ and

$$\epsilon_e \sim mc^2\mu^{3/4}\sqrt{\frac{mc^2}{\hbar\omega}}, \qquad t_e \sim \frac{\hbar}{\alpha mc^2\mu^{1/4}}\sqrt{\frac{mc^2}{\hbar\omega}}. \tag{5}$$

Since the energy of a photon emitted by an ultrarelativistic positron with $\chi_e \gtrsim 1$ is of the order $\epsilon_\gamma \sim \epsilon_e$, $\chi_\gamma \sim \chi_e$, the photon lifetime with respect to pair production t_γ is of the order t_e.

It can be easily verified from Eqs. (3) and (5) that the following hierarchy of time scales

$$t_{\rm acc} \lesssim t_e, t_\gamma \ll t_{\rm esc} \tag{6}$$

is respected if

$$\mu \gtrsim 1, \qquad \mu^{1/4} \gg \frac{1}{\alpha}\sqrt{\frac{\hbar\omega}{mc^2}}. \tag{7}$$

Equation (6) determines the necessary conditions for occurrence of electromagnetic cascade. Indeed, if $t_e \gtrsim t_{\rm acc}$, then χ_γ of the emitted photon can be $\gtrsim 1$ and hence it may create a pair. On the other hand, the number of successive events of photon emission and pair production throughout the time period $t_{\rm esc}$ is large if $t_e, t_\gamma \ll t_{\rm esc}$.

It is worth noting that for optical frequencies the second restriction in Eq. (7) is weaker than the first one. Therefore, the necessary conditions for occurrence of the cascade is reduced to the relation $\mu \gtrsim 1$ in this case. Consequently, the field strength E_* determines a natural threshold for electromagnetic cascades. Such field performs the work $\sim mc^2$ over an electron at its radiation free path. The corresponding laser intensity is $I_* \approx 2.5 \times 10^{25}$ W/cm^2. Hence, the cascade will occur at intensities of the order of 10^{25} W/cm^2 or higher.

The total number of pairs created per one laser shot can be estimated as

$$N_e \sim \exp\left(\frac{t_{\rm esc}}{t_e}\right) \sim \exp\left[\pi\alpha\mu^{1/4}\sqrt{\frac{mc^2}{\hbar\omega}}\right], \tag{8}$$

see the solid curve in Fig. 2 [24].

The most important result of our estimation is that the number of created pairs is so huge that their net energy can become even equal to the energy stored in the laser pulse. Under assumption that the laser pulse is focused to diffraction limit, so that the volume of the focal area where pairs are created is $\sim(\lambda/2)^3$, the total energy of laser field can be estimated by $W \sim (E^2/4\pi)(\lambda/2)^3$. Thus, the maximum number of pairs that can be created is restricted by the value

$$N_{e,\max} = \frac{W}{2\epsilon_\epsilon} \sim \alpha\mu^{5/4}\left(\frac{mc^2}{\hbar\omega}\right)^{5/2}. \tag{9}$$

This quantity is represented by a dash line in Fig. 2. We see that the net energy of created pairs becomes of the order of the energy of the laser pulse already at $\mu \approx 10$. Such value

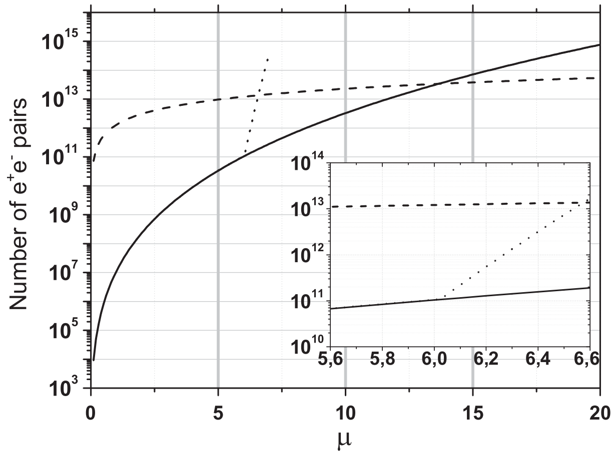

FIG. 2. Pair production as a function of μ. The solid curve corresponds to the number N_e of pairs produced by a single cascade process. The dotted curve shows the number of pairs produced by multiple cascades generated by pairs created by two colliding circularly polarized 10 fs laser pulses. The branching point corresponds to the threshold value of μ where the spontaneous pair production begins. The dash line shows the limit for N_e determined by the energy of the laser pulse. The laser frequency $\hbar\omega = 1$ eV. The inset shows the magnified region of intersection of the curves.

of μ will be attained in collision of two circularly polarized laser pulses of intensity $I \approx 6 \times 10^{26}$ W/cm^2.

This estimation was obtained under assumption that only one pair was created per one shot. However, according to Ref. [9] the threshold intensity for pair creation by two colliding circularly polarized 10 fs laser pulses with $\hbar\omega = 1$ eV is equal to $I_{\mathrm{th}} \approx 2.3 \times 10^{26}$ W/cm^2. The dependence of the number of created pairs N_e on the intensity of colliding pulses is very sharp, and at $I \approx 6 \times 10^{26}$ W/cm^2 it reaches the value of $N_e \approx 6 \times 10^8$. This means that destruction of the laser pulse will take place much earlier than discussed above. Dependence of the total number of created pairs with due regard for the number of pairs initially created by the laser field is presented by dot line in Fig. 2. The branching of the solid and dot curves takes place at the threshold value of $\mu = \mu_{\mathrm{th}}$ which is equal approximately to $\mu_{\mathrm{th}} \approx 6$ for the case of two colliding circularly polarized laser pulses according to Ref. [9]. It is assumed that for $\mu < \mu_{\mathrm{th}}$ a particle initiating the cascade was not created due to the pair creation process but was situated in the field initially. It is worth emphasizing that, as opposed to the solid line, the results represented in Fig. 2 by the dot line are valid only for the case of two colliding circularly polarized laser pulses.

We see that N_e reaches the maximum possible value at $\mu \approx 6.6$ which corresponds to intensity of colliding pulses $I \approx 2.7 \times 10^{26}$ W/cm^2.

To conclude, we have described a new kind of QED cascades that can be initiated in the laser focus by even a single pair created by the laser field from vacuum. The multiplicity of such cascade is no longer limited by the initial energy of a particle initiating it. Rather, it is limited by the time of stay of particles in laser field, or, under more extreme conditions, by the total electromagnetic energy stored in the pulse. In the latter case, development of the cascade causes depletion of the laser pulse, thus imposing strict limitation on the maximum attainable laser intensity.

In this letter, we have based our consideration on a model of rotating electric field. To obtain more realistic results one needs solve the system of 3D kinetic equations taking into account back-reaction of the cascade development on the laser field. At the moment, we are working on this program. However, our present estimations convincingly confirm Bohr's 80-years-old conjecture that the critical QED field strength E_S can be never attained for a pair creating electromagnetic field. Note, that our conclusion applies to the electric field strength in the reference frame, where the magnetic field either vanishes or $\boldsymbol{H} \parallel \boldsymbol{E}$; see [8].

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