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# Nonlinear vibrations of steelpans: analysis of mode coupling in view of modal sound synthesis.

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## ABSTRACT

Steelpans are musical percussions made from steel barrels. During the manufacturing, the metal is stretched and bended, to produce a set of thin shells that are the differents notes of the instrument. In normal playing, each note is struck, and the sound reveals some nonlinear characteristics which give its peculiar tone to the instrument. In this paper, an experimental approach is first presented in order to show the complex dynamics existing in steelpan's vibrations. Then two models, based on typical modal interactions, are proposed to quantify these nonlinearities. Finally, one of them is observed in free oscillations simulations, in order to compare the internal resonance model to the steelpan vibrations behaviour in normal playing. The aim is to identify the important modes participating in the vibrations in view of building reduced-order models for modal sound synthesis.

## 1. INTRODUTION

Steelpans belong to a musical instruments family coming from the island of Trinidad and Tobago. They are usually played in steelbands (see Fig. 1(a)), that are orchestras composed of steelpans covering a range of several octaves.

A steelpan is a tuned percussion, built from cylindrical steel barrels subjected to several stages of metal forming that stretch and bend the structure. The top of the barrel is pressed, hammered, punched and burnt in order to obtain a sort of main thin bowl within which convex substructures are formed. Each convex dome corresponds to a musical note, which natural frequency is precisely tuned according to harmonic relationships ( $f$ , $2f$ , 3 (or 4) $f$, ...). Usually, this instrument is played by striking each note with a stick covered by a piece of rubber, as shown on the Fig. 1(b). When a note is stroke with a stick, vibration amplitudes are such that geometric nonlinearities cannot be neglected, and are recognized as a key feature for explaining the peculiar tone of the steelpan [1]. This nonlinearity combined with harmonic relationships, due to steelpan tuning, can activate internal resonances creating strong energy transfers between eigenmodes.

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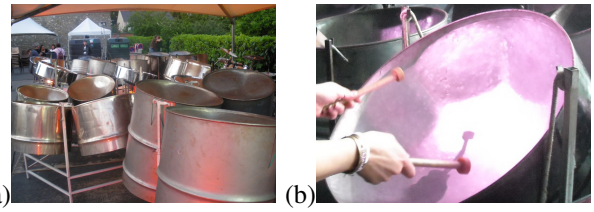


**Figure 1**. Pictures of steelpans : (a) Steelpans of a typical steelband. (b) Steelpan in normal playing.

Vibrations modelling have been proposed in a series of papers by Achong et al. [2]. In these works, the steelpan is considered as a nonlinear system of oscillators, and energy transfers between normal modes of vibration are highlighted. Rossing et al. [3, 4] have performed modal analyses by holographic techniques to observe modal interactions between harmonically tuned notes. More recently, numerical modal analyses with the finite element methods have been proposed [5]. The steelpans sound radiation has been recently addressed experimentally by Copeland et al. [6]. Finally, some metallurgical issues during the steelpan making have been considered in [7].

In this contribution, we propose a refined analysis of modal couplings and energy exchanges occurring in nonlinear vibrations of steelpan. A modal analysis first reveals the appearance of pairs of modes from the second harmonic, for each note of the pan. These pairs of modes, having nearly equal eigenfrequencies, are interpreted as a consequence of the localization of the vibration into the notes area.
Secondly, experiments in forced vibrations allow to reveal the complex nature of the energy exchanges between the modes, that are excited for very small levels of vibration amplitudes. Simple original models including a 1:2:2 and a 1:2:4 resonance are then fitted to the experiments, showing that: (i) mode pairing substantially complexifies the dynamical behaviours and favours the appearance of unstable regimes, (ii) few details of the resonance curves are missed by those 3-dofs models, indicating that even at very small amplitudes of vibrations, a complex dynamics involving more than 3 modes is at hand.
Those findings are used to derive oscillator models for sound synthesis. The first results with three dofs show that the main features (energy transfers and enveloppe modulation) are recovered.

## 2. STEELPAN TUNING AND LINEAR ANALYSIS

In steelpan making, a fine tuning of the notes already formed is performed at the end of the building process. Most of the time, the steelpan maker tunes the first three harmonics of each note [8]. He begins with the fundamental frequency (the pitch) by modelling the center of the note. Then, he focuses on the overtones. He adjusts the frequency of two upper harmonics (partials) by modelling the periphery of the note area, as it is shown of Fig. 2(a). This procedure can be easily understood by considering that each harmonic of a given note is associated to a particular vibration mode shape, as it will be explained in the following.

Modal analysis is usually used to characterize the linear behaviour of a structure by identifying eigenfrequencies, mode shapes and modal damping coefficients. A home-made non-contact coil/magnet exciter is used to excite the steelpan at a given point. The equivalent point force is estimated by recording the current intensity in the coil [9]. The steelpan vibratory response, in velocity, is measured with a laser vibrometer.

The steelpan, shown on the figure 2(b), is a right barrel of a double second (middle-high frequency steelpan). It is composed of 19 precisely tuned notes, distributed on three concentric circles, the lower notes being on the outer circle. Previous studies have shown that the vibration remains confined around the solicitation area [3]. Hence, the scan is more particularly focused on G3 (of fundamental frequency $f_1$ ) and its harmonically tuned neighbours G4 ($2f_1$) and G5 ($4f_1$).

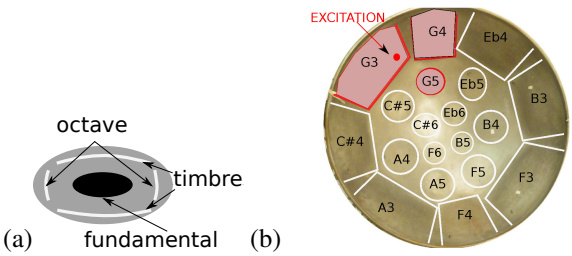



**Figure 2**. (a) Steelpan tuning. (b) Modal analysis of the steelpan used for the experiment (right barrel of a double second) excited on G3.

Fig. 3 shows the transfer function, measured on the excitation point, in the frequency range [0, 1700] Hz, and the associated mode shapes of the structure. One can see that the first three modes are perfectly tuned like $f_1$, $2f_1$ and $4f_1$, while the fourth and the fifth departs a little from the perfect harmonic relationship, and are slighlty shifted from the exact $6f_1$ and $8f_1$ relation.
More precisely, around $f_1$, the modal shape of the structure is focused on the excited note only. Around $2f_1$, a double peak is clearly visible indicating that the mode is degenerate with two mode shapes around the same frequency. The first one has for eigenfrequency $f_2 = 390$ Hz and is composed of the second vibration pattern of the G3 note together with the fundamental vibration mode of the G4 note. The second one has its eigenfrequency at $f_3 = 397.8$ Hz and its mode shape is similar except the fact that the pattern on the G4 note is out of phase. Finally the measurement reveals also that at $4f_1$, two degenerate modes are also at hand, with eigenfrequencies $f_4 = 789.5$ Hz and $f_5 = 799.3$ Hz, and companion mode shapes.

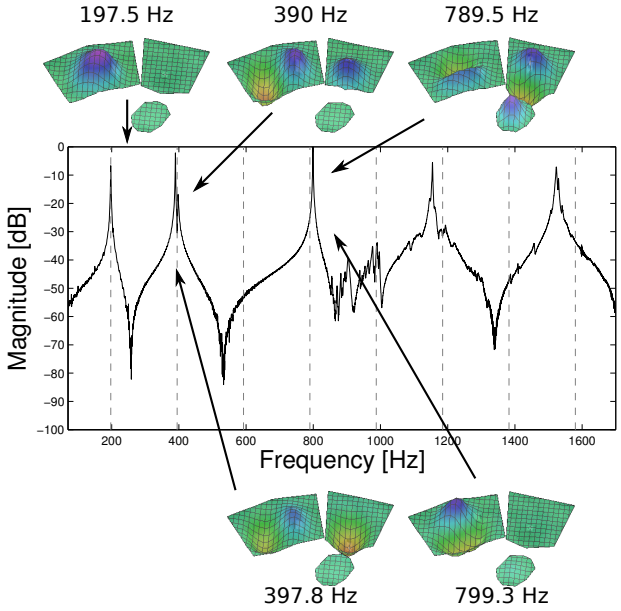



**Figure 3**. Modal analysis of the steelpan excited on the note G3. FRF measurement and associated mode shapes.

This modal analysis reveals that in steelpan vibrations, modes appear by parts from the second partial. This is a consequence of the fact that vibrations are strongly localized into notes areas, and is a classical feature in mode localization that has been observed for instance in simple beam systems [10].

Forced vibrations at higher force amplitudes will now be detailed to depict how energy is transferred between these modes.

## 3. FORCED OSCILLATIONS

The previous linear analysis shows that the first frequencies of the steelpan note are organized as a 1:2:2:4:4 sequency. In a nonlinear regime, and in the case of thin curved structures, harmonic relationships create strong modal interactions via internal resonances [11]. In order to investigate the nonlinear behaviour of the steelpan, the same home-made coil/magnet exciter is used, but higher force amplitudes are applied. The coil/magnet exciter has been thoroughly analyzed for calibration in [12], where it has been shown that for the vibration amplitudes encountered, harmonic distortion is less than 1%, ensuring a clean and reproducible harmonic excitation. An external sinusoidal current is applied with an external excitation frequency around the first eigenfrequency ($f_{dr} \simeq f_1$). The vibratory response of one point of the excited note is measured. The first three harmonics (at $f_1$, $2f_1$ and $4f_1$) of the response

are precisely measured. Then, two analytical models of internal resonance relationships are proposed to fit the experiment and identify some modal interactions in steelpans vibrations. More details can be found in [13, 14].

### 3.1 Experiments

In Fig. 4 the nonlinear response of the steelpan excited around 197.5 Hz with $I = 5$A is shown. The harmonics 1, 2 and 4 of the recorded displacement are shown, they are denoted respectively by $w_{H1}$, $w_{H2}$ and $w_{H4}$. A strong coupling between these harmonics, oscillating at $f_1$, $f_2 \simeq 2f_1$ and $f_3 \simeq 4f_1$, is revealed. Markers are inserted into the figures to precisely locate, in frequency, the different eigenfrequencies of the system.
First, a 1:2:2 internal resonance is observed through the complex behaviour of the measured amplitude at twice the excitation frequency. The energy transfered gives a large amplitude of vibration to the second (390 Hz) and the third eigenmode (789.5 Hz). Secondly, coupling with the fourth harmonic is also observed where $w_{H4}$ seems to be slaved to $w_{H2}$ due to the similar global shapes of the two corresponding amplitudes. One can assume that a 1:2:2:4 resonance is here activated.
A strong peak of amplitude exists around $f_{dr} \simeq 192$ Hz. At this frequency, the first mode, directly excited, reaches $0.06$mm, the second harmonic $0.08$mm and the fourth one $0.003$mm. Considering the thickness of a typical steelpan initial barrel around 1 mm, one can conclude that geometric nonlinear effects are exhibited for vibration amplitudes of 1/16 times the thickness.
Then, a quasiperiodic regim, around 193 Hz, reveals a complex dynamics of the system. This implies that there is a difference between the forward and the backward excitation. A jump phenomenom is observed at 192.6 Hz.
Compared to the theory [14, 15], the shapes of the solutions are slightly different than a 1:2 or a 1:2:2 internal resonance, probably because the cubic nonlinearity is not anymore negligible. An other remark on the effects of cubic nonlinearities can be the shift of the curves to the left part of the graph, compared to the linear initial frequencies. The maximum of each harmonic response is observed around $f_{dr} \simeq 192$ Hz.

### 3.2 3-dofs internal resonance models fitting

The complicated dynamics exhibited by the forced vibrations is now modeled by two simple systems involving three internally resonant modes, so as to highlight the most salient features of the dynamics of the steelpan. The two models involve either a 1:2:2, or a 1:2:4 eigenfrequency relationship. For these 3-dofs models, analytical solutions are accessible via multiple scales analysis [14]. Model fitting to experimental measurements will thus shed new light on the identification of nonlinear coupling coefficients as well as energy transfers.

Considering the steelpan as a curved thin structure, the transverse displacement $w$ can be discretised by expanding

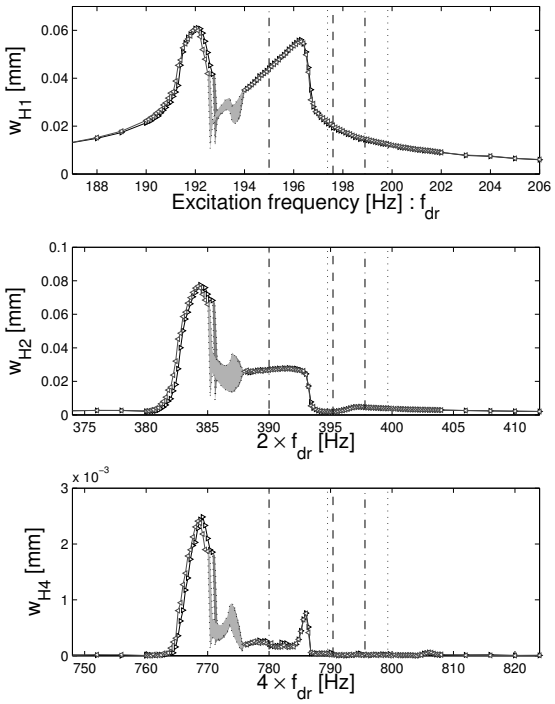



**Figure 4**. Frequency response curves $I = 5$A, excitation frequency $f_{dr}$ in the vicinity of the first mode. Perlines: harmonics 1, 2 and 4 of the measured displacement $w$. Forward (black line: $\triangleright$) and backward (grey line: $\triangleleft$) frequency sweeps. Linear eigenfrequency markers $f_1$ (– –) ; $2f_1$ (– · –) and $4f_1$ ($\cdots$).

$w$ on the linear modes basis $\{\Phi_k\}$ as

$$w(\mathbf{x}, t) = \sum_{k=1}^{N} \Phi(\mathbf{x}) q_k(t), \tag{1}$$

where $q_k$ is the modal coordinate and $\Phi_k$ is the mode shape amplitude value at the spatial point $\mathbf{x}$.

Now two 3-dofs simple models are proposed, considering the most important coupling found in the modal analysis, and thus displaying respectively a 1:2:2 and the 1:2:4 internal resonance. The 1:2:2 internal resonance considers three modes which eigenfrequencies are tuned such that $\omega_3 \simeq \omega_2 \simeq 2\omega_1$. The associated normal form of the amplitude of the three corresponding modes reads :

$$\begin{cases} \ddot{q}_1 + \omega_1^2 q_1 = \varepsilon \left[ -2\mu_1 \dot{q}_1 - \alpha_1 q_1 q_2 - \alpha_2 q_1 q_3 + F_1 \cos \Omega t \right], \\ \ddot{q}_2 + \omega_2^2 q_2 = \varepsilon \left[ -2\mu_2 \dot{q}_2 - \alpha_3 q_1^2 \right], \\ \ddot{q}_3 + \omega_3^2 q_2 = \varepsilon \left[ -2\mu_3 \dot{q}_3 - \alpha_4 q_1^2 \right], \end{cases} \tag{2}$$

where $\omega_k = 2\pi f_k$ denotes the angular frequency of the mode $k$ and $\mu_k$ its damping coefficient. $\Omega = 2\pi f_{dr}$ is the external frequency and $F_1$ the amplitude of the external force. In first approximation, only quadratic nonlinear coupling terms $\alpha_i$ are kept. Only four of them are present. They correspond to the resonant terms. All other possible nonlinear quadratic terms have no importance for the global dynamics and can be cancelled. Nonlinear coefficients, damping terms and external forcing are assumed to

be small as compared to the linear oscillatory part, and thus are scaled by $\epsilon \ll 1$. According to the multiple scales development, the solutions of the dynamical system Eqs. (2) are obtained as:

$$\begin{cases} q_1 = a_1 \cos\left(\Omega t + \varphi_1\right), \\ q_2 = a_2 \cos\left(2\Omega t + \varphi_2\right), \\ q_3 = a_3 \cos\left(2\Omega t + \varphi_3\right), \end{cases} \tag{3}$$

where $a_k$ and $\varphi_k$ are amplitudes and phases of the solution $q_k$, respectively. In term of transverse displacement, the equation (1) leads to $w(\mathbf{x},t) = w_1 \cos(\Omega t + \varphi_1) + w_2 \cos(2\Omega t + \gamma_2)$, where $w_1 = \Phi_1 a_1$ and where $w_2$ and $\gamma_2$ are combinations of $\Phi_2$, $\Phi_3$, $a_2$, $a_3$, $\varphi_2$ and $\varphi_3$.

The 1:2:4 internal resonance, is a 3-dofs nonlinear dynamical system constructed with the following eigenfrequency relationship: $\omega_3 \simeq 2\omega_2 \simeq 4\omega_1$. It reads:

$$\begin{cases} \ddot{q}_1 + \omega_1^2 q_1 = \varepsilon\left[-2\mu_1\dot{q}_1 - \alpha_5 q_1 q_2 + F_1 \cos\Omega t\right], \\ \ddot{q}_2 + \omega_2^2 q_2 = \varepsilon\left[-2\mu_2\dot{q}_2 - \alpha_6 q_1^2 - \alpha_7 q_2 q_3\right], \\ \ddot{q}_3 + \omega_3^2 q_3 = \varepsilon\left[-2\mu_3\dot{q}_3 - \alpha_8 q_2^2\right], \end{cases} \tag{4}$$

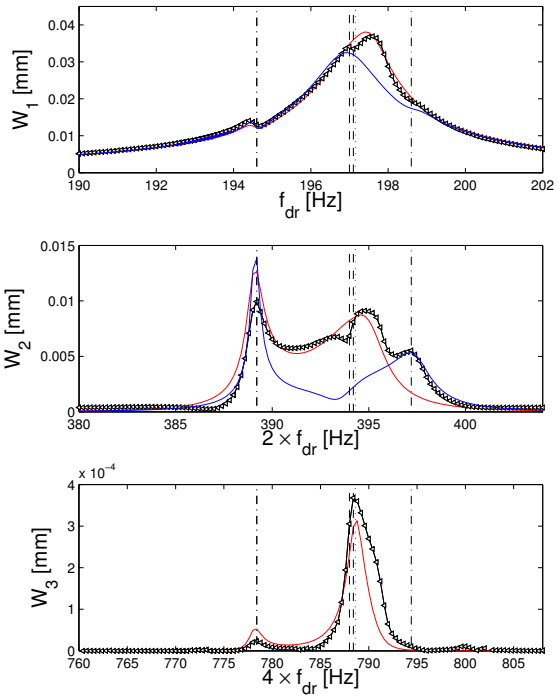
where $\alpha_{5,6,7,8}$ are the four new nonlinear coupling coefficients associated to the 1:2:4 internal resonance system. Solutions of Eqs. (4) are:

$$\begin{cases} q_1 = a_1 \cos\left(\Omega t + \varphi_1\right), \\ q_2 = a_2 \cos\left(2\Omega t + \varphi_2\right), \\ q_3 = a_3 \cos\left(4\Omega t + \varphi_3\right). \end{cases} \tag{5}$$

In that case, two upper-harmonics, oscillating at $2\Omega$ and $4\Omega$, simultaneously appear. The transverse displacement is $w(\mathbf{x},t) = w_1 \cos(\Omega t + \varphi_1) + w_2 \cos(2\Omega t + \phi_2) + w_3 \cos(4\Omega t + \phi_3)$, where $w_1 = \Phi_1 a_1$, $w_2 = \Phi_2 a_2$ and $w_3 = \Phi_3 a_3$.

Fig. 5 presents the 1:2:2 model fitting (in red color), first compared to the experiment, then compared to a 1:2:4 internal resonance model fitting. Experimental fitting, presented in Fig. 5, determines the value of each nonlinear coefficient while the linear parameters ($\omega_k$, $\mu_k$) are identifyed by modal analysis. The amplitude of the external current is $I = 2$A. The two 3-dofs models allow to locally identify some modal interactions in steelpan vibrations. Each of them locally fits parts of the experimental curves. Markers of linear frequencies indicate the mode activated by nonlinear coupling. This fitting shows that simple models can be used to enhance the comprehension of the complicated dynamics experimentally observed. More complicated scenarii invoking also the presence of the modes at $6f_1$ and $8f_1$ may also be activated in certain vibratory regimes.

The steelpan is harmonically tuned (both between notes and between harmonics in a note). Therefore, the thinness of the curved structure and the amplitude of sollicitation of the note lead to nonlinear behaviour with modal interactions and energy transfers, consequently. Internal resonance models displaying a 1:2:2 and a 1:2:4 frequency relationships allow to recover the main features of the FRFs,




| | Mode 1 | Mode 2 | Mode3 |
|---|---|---|---|
| $F_k = \omega_k/2\pi$ [Hz] | 197.1 | 389.2 | 788.6 |
| $\xi_k = \mu_k/\omega_k$ [%] | 0.0045 | 0.001 | 0.001 |
| $\Phi_k(\mathbf{x})$ | 0.295 | 0.339 | 0.06 |

**Figure 5**. Frequency response curves and experimental fitting ($I = 2$A) (black line). Nonlinear coupling coefficients identified: 1:2:2 internal resonance model (blue line) - $\alpha_1 = 0.42$, $\alpha_2 = 0.46$, $\alpha_3 = 0.46$, $\alpha_4 = 0.39$ and 1:2:4 internal resonance model (red line) - $\alpha_5 = 0.24$, $\alpha_6 = 0.34$, $\alpha_7 = 0.88$, $\alpha_8 = 0.53$.

while non-modelled effect appears to be easily interpreted. The most complete model for that case should be a 1:2:2:4:4 one, unfortunately analytical solutions for that problem are not tractable.

## 4. NONLINEAR MODEL FOR SOUND SYNTHESIS

### 4.1 Free oscillations measurement

In Fig. 6, two time-frequency representations of free oscillations of the G3 note in normal playing are shown. A weak stroke (see Fig. 6(a)) reveals that most of energy is stored within the first two harmonics. Comparatively, a strong stroke (see Fig. 6(b)) has much more energy on the higher frequencies. It can be noticed that, on these two spectrograms, there is no energy at $F = 3f_1$. For the low frequency range, only the tuned harmonics are excited when the steelpan is played.

Fig. 7 represents the measurements of the Fig. 6 filtered on the harmonics 1, 2 and 4 (denoted $H_1$, $H_2$, $H_4$), showing the evolution with the time of the amplitude of each

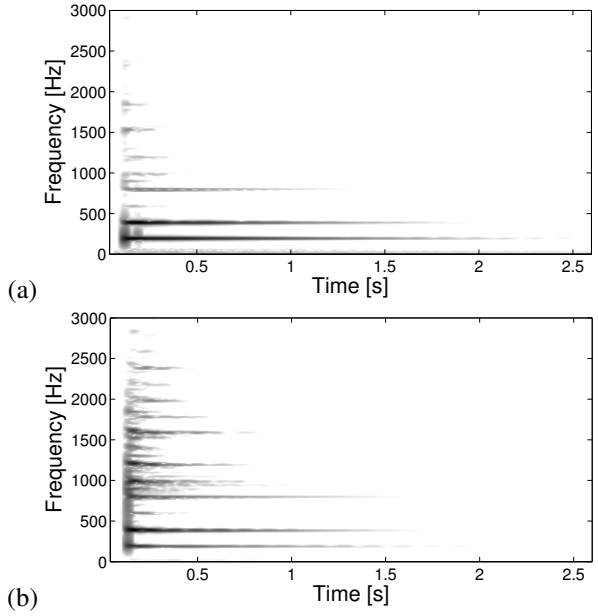



**Figure 6**. Experimental spectrograms of two free oscillation measurement with two stick-impact intensities. (a) Weak stroke. (b) Strong stroke.

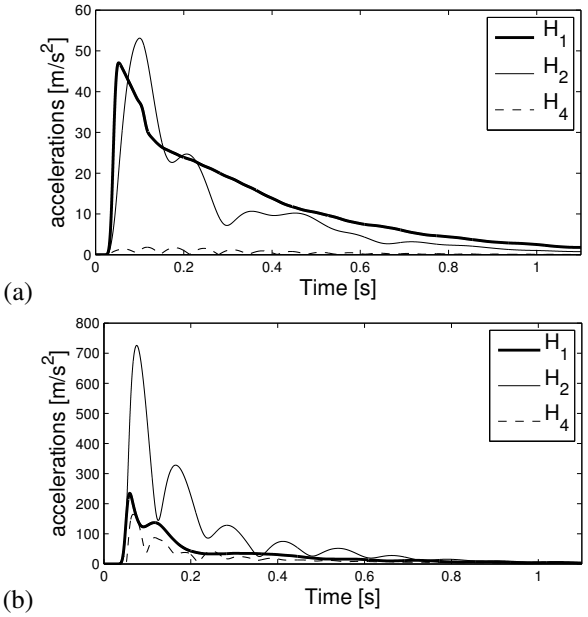



**Figure 7**. Same measurements than Fig. 6, filtered on harmonics 1, 2 and 4. Evolution of each harmonics with time.

harmonic. First, on the Fig. 7(a), it can be observed that the second harmonic reaches its maximum later than the first one does. Then, the amplitude of the maximum is a little bit higher than the first one. The fourth harmonic presents a very small amplitude. For a strong stroke (see Fig. 7(b)), the amplitude of the second harmonic is much larger than the first one. It can be deduced that energy is transfered from the first mode to the second one, and the intensity of the transfer depends on the intensity of the excitation. The fourth one has much more energy than in the first case. Its amplitude is very close to the first harmonic. Thus, it is shown that initial conditions are very important in the steelpan response.

### 4.2 Free oscillations simulation

The 1:2:4 adjusted model is now analyzed in free oscillations behaviour. The ordinary differential equation solver (ODE45) of Matlab is used. The linear parameters and the nonlinear coupling coefficients values are deduced from the experimental fitting in forced oscillations (see Fig. 5). An initial condition in displacement is given so as to mimic a delta dirac temporal excitation.

Fig. 8 shows the result of two simulations. The first one (Fig. 8(a)) is for a weak initial condition ($w_0 = 0.15$ mm) and the second one (Fig. 8(b)) is for a larger one ($w_0 = 0.65$ mm). The acceleration of the three transverse displacements ($w_1$, $w_2$ and $w_3$) are represented versus time. Compared to the filtered measurement (Fig. 7), the global time evolution of a steelpan sound is recovered for the two initial conditions. The right amplitude ratios between modes are observed. Also, one can remark that the period of the oscillations is qualitatively the same in the experiments and the simulations.

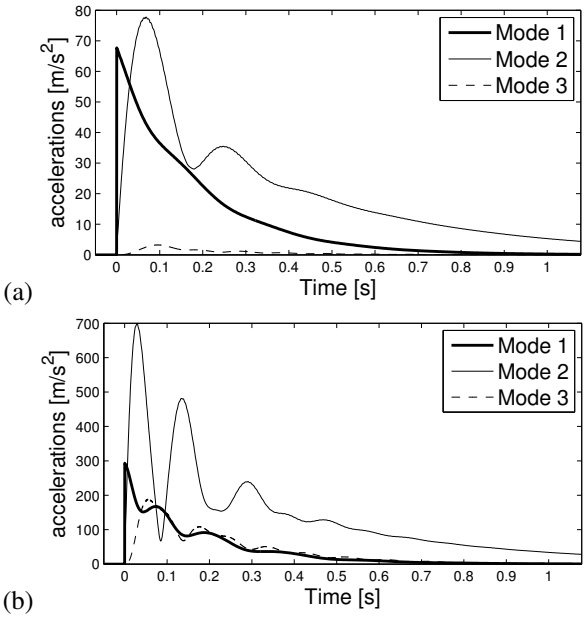



**Figure 8**. 1:2:4 internal resonance model in free oscillations simulation. Initial conditions in displacement: (a) $w_0 = 0.15$ mm, (b) $w_0 = 0.65$ mm.

These time simulations confirm that with regard to the global dynamics carried by the envelope modulation of the main harmonics, the 3-dofs model allows to recover the most salient nonlinear features, and are thus identified as a key component in steelpans's sound and vibrations.

## 5. CONCLUSION

Nonlinear vibrations of a steelpan presenting a 1:2:4 modal tuning ($f$, $2f$, $4f$) have been investigated thanks to a refined modal analysis, measurements of frequency-response curves in forced oscillations and time domain simulations in free, impacted vibrations, using 3-dofs models identified from the forced response.

The main outcomes of the present study are the following

- The modal analysis has clearly evidenced the fact that modes appear by pairs from the second harmonic of each note, a feature that is only scarcely mentioned in the literature. This pairing is interpreted as a consequence of the strong localization of the vibrations into the notes area. Indeed, simpler systems consisting of beams with stiffness imperfections reveals the same degeneracy which is thus generic for structural systems having at least two minima of stiffness [10].

- A consequence of this mode pairing is that the dynamical equations are complexified. Analytically, one can also show that the appearance of 1:2:2 (instead of simpler 1:2) or *e.g.* 1:2:2:4:4 (instead of simpler 1:2:4) internal resonances in a nonlinear systems favour instabilities [14].

- The forced response analysis reveals the complex dynamics of steelpans with appearance of energy exchange and quasiperiodic regimes for very small amplitudes of vibration.

- 3-dofs models that are analytically solvable have been fitted to measured resonance curves, showing that the main features are recovered by considering 1:2:2 and 1:2:4 resonances. However small details in the resonance curves are not fitted, advocating for the fact that the precise dynamics is more complex with energy exchange between pairs of modes and thus solutions comprising *e.g.* 1:2:2:4:4 resonances and even for some amplitudes 1:2:2:4:4:6 and 1:2:2:4:4:8.

- The temporal simulations allow recovering the main features of the energy exchange, once again confirming that the most important part of the dynamics is carried by the identified resonances. However, small details are still missing to the ear for a sound synthesis approach, stating clearly that the higher modes (at 6f and 8f) have to be included in a simple modal model for sound synthesis.

At the conference, sound synthesis will be presented in order to more precisely quantify the importance of the higher modes for the sound production, as well as the impact model.

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