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A phenomenological model for predicting the effect of damping on wave turbulence spectra in vibrating plates

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Abstract Thin plates vibrating at large amplitudes may exhibit a strongly nonlinear regime that has to be studied within the framework of wave turbulence. Experimental studies have revealed the importance of the damping on the spectra of wave turbulence, which precludes for a direct comparison with the theoretical results, that assumes a Hamiltonian dynamics. A phenomenological model is here introduced so as to predict the effect of the damping on the turbulence spectra. Self-similar solutions are found and the cut-off frequency is expressed as function of the damping rate and the injected power.

The large amplitude vibrations of thin plates are well described by the Wave Turbulence (WT) theory. The out-of-equilibrium solutions are found from the kinetic equations [1], which admit two different solutions [2]:

- The so-called Rayleigh-Jeans solution where energy is equiparted along all possible lengthscales. In this case the the density of energy $E^{\mathcal{R}J}(\omega)$ is a constant with respect to the frequency.
- The Kolmogorov-Zacharov (KZ) solution with an energy flux cascading from the largest to the smallest wavelength. In this case the density of energy reads :

$$E^{\mathcal{K}Z}(\omega) = A\varepsilon^{\frac{1}{3}}\log^{\frac{1}{3}}\left(\frac{\omega^\star}{\omega}\right), \tag{1}$$

  with A a constant, ε the (conserved) energy flux and $\omega^\star$ a cut-off frequency.

Numerous experiments have underlined the effect of damping [3,4]. In order to study its effect on the WT spectrum, a phenomelogical model is introduced, which describes the temporal and frequency dependence of the energy density $E(\omega)$. It reads:

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial \omega}(\omega E^2 \frac{\partial E}{\partial \omega}) - \hat{\gamma}E, \tag{2}$$

For $\hat{\gamma} = 0$ (conservative case), the phenomenological model retrieves stationary (RJ and KZ spectra) as well as non-stationary solutions of the kinetic equation [5].

Fig. 1(a) shows the stationary spectra of turbulence obtained from Eq. (2) for a damping law of the form $\hat{\gamma} = \xi\omega^{0.6}$, with relative values of ξ (with respect to the smallest one) ranging from 1 to 5. Interestingly the solutions are self-similar and collapse on a single curve when rescaled by the cut-off frequency ω_c, as shown in Fig. 1(b). This solution is different from the KZ spectrum for the conservative case –shown as a green dashed line in Fig. 1(b)– showing undoubtedly the effect of the damping.

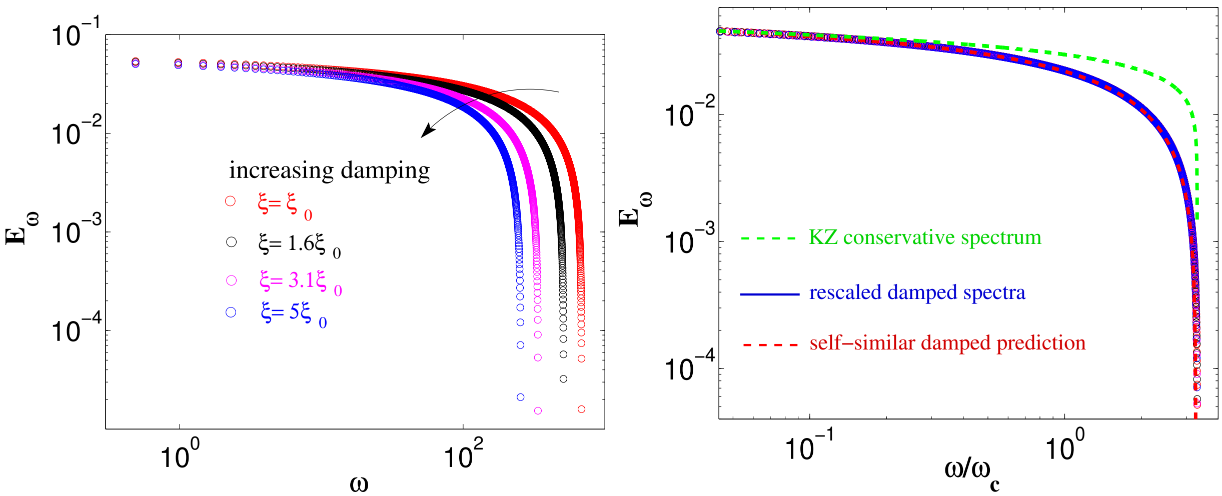

Figure 1: (a) Stationary energy spectrum $E(\omega)$ for increasing values of the damping. (b) Energy spectra $E(\omega)$ as functions of the rescaled frequency ω/ω_c .

Let us consider now a general damping rate expressed as $\hat{\gamma} = \xi\omega^{\lambda}$, where the exponent λ controls the frequency dependence of the losses and ξ its amplitude. The self-similar solution for the spectrum can be expressed as:

$$E(\omega) = \varepsilon_I^{1/3} f_\eta \left(\frac{\omega}{\varepsilon_I^y \xi^z} \right), \qquad (3)$$

with $z = -\frac{1}{1+\lambda}$, $y = \frac{2}{3(1+\lambda)}$, and f_η solution of the differential equation:

$$\partial_\eta(\eta f_\eta^2 \partial_\eta f_\eta) - f_\eta \eta^\lambda = 0, \qquad (4)$$

The solution for this differential equation is shown in Fig. 1(b) as a red dashed line and matches exactly with the direct simulation. Finally one is able to predict the cut-off frequency as function of the damping, and the injected flux as :$\omega_c = \varepsilon_I^{\frac{2}{3(1+\lambda)}} \xi^{-\frac{1}{1+\lambda}}$, which constitutes the main result of the present study.

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