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High-order absorbing boundary conditions with edge and corner compatibility for the Helmholtz equation

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Abstract

We deal with the finite element solution of 3D time-harmonic acoustic wave problems defined on unbounded domains, but computed using cuboidal computational domains with artificial boundaries. We combine a standard finite element method for the Helmholtz equation with high-order absorbing boundary conditions *(on the faces of the domain)* and compatibility relations *(on the edges and the corners)* that provide an arbitrary high accuracy.

Key words: Helmholtz equation, Finite element method, Radiation boundary condition, Corner compatibility
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The finite element methods must be combined with specific boundary techniques, such as perfectly matched layers and non-reflective boundary conditions, to accurately simulate the radiation of wavefields at the artificial boundaries of truncated computational domains. Padé-type high-order absorbing boundary conditions (HABCs) can provide an arbitrary high accuracy [2], but they are generally limited to regions with smooth boundaries. We propose a comprehensive strategy for cuboidal domains, with treatments for the edges and the corners.

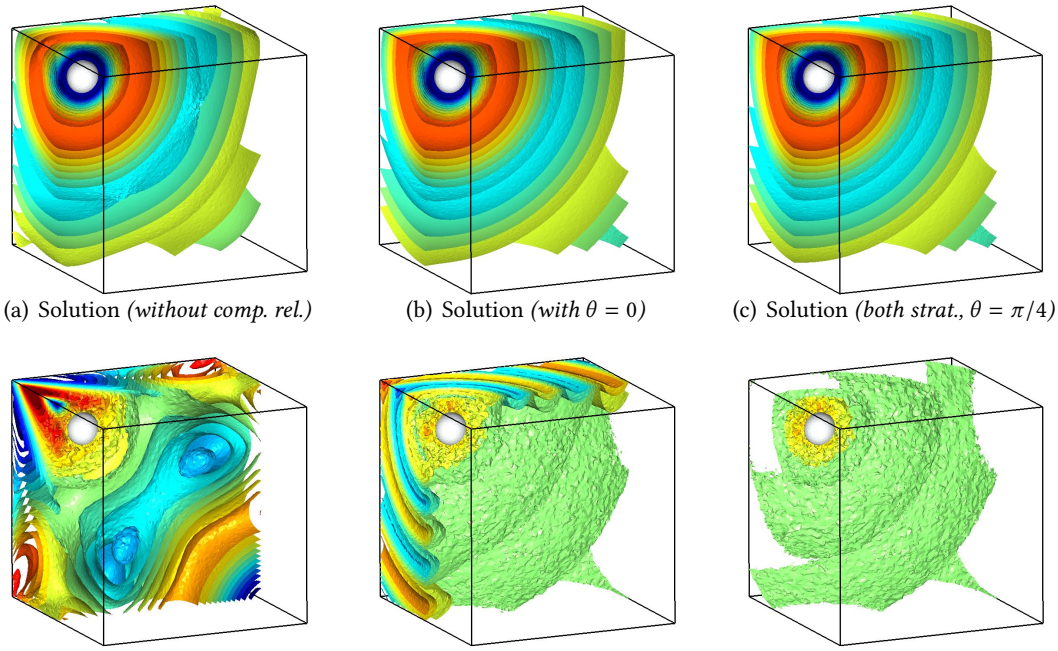
Let the field $u(\mathbf{x})$ governed by the Helmholtz equation $\Delta u + k^2 u = 0$ on the cube $\Omega = [-L, L]^3$. On the faces of Ω, we consider the HABC obtained by approximating the square root in the exact non-reflective boundary operator thanks to the $(2N+1)^{\text{th}}$-order Padé expansion with a θ-rotation of the branch cut [2]. On the face belonging to the plane $x = L$, the HABC can then be written as

$$\partial_x u = \imath k e^{\imath\theta/2}\left(u + \frac{2}{M}\sum_{n=1}^{N} c_n\,(u + u_n)\right), \quad \text{with } c_n = \tan(n\pi/M) \text{ and } M = 2N+1. \tag{1}$$

This condition involves N auxiliary fields u_n defined only on the face and governed by

$$k^2\left(e^{\imath\theta}c_n + 1\right)u_n + k^2 e^{\imath\theta}\,(c_n + 1)\,u + \left[\partial_{yy} + \partial_{zz}\right]u_n = 0, \quad n = 1\ldots N. \tag{2}$$

Because of the spatial partial derivatives in Eq. 2, boundary conditions must be prescribed on the boundary of the face (*i.e.* on the edges of the cube) for each auxiliary field u_n.



(a) Solution *(without comp. rel.)* (b) Solution *(with $\theta = 0$)* (c) Solution *(both strat., $\theta = \pi/4$)*

(d) Error *(without comp. rel.)* (e) Error *(with $\theta = 0$)* (f) Error *(both strat., $\theta = \pi/4$)*

Figure 1: Solutions and errors for the benchmark with HABCs ($N = 4$). Three cases are shown: without compatibility relations *(left)*, without rotating branch cut *(middle)*, with both strategies *(right)*. Isosurfaces of the solutions (scale: [-0.2,0.2]) and the errors (scale: [-0.015,0.015]) are represented in only half the domain. We use a mesh made of approx. 10^6 tetrahedron and P1 elements.

In our strategy, we introduce new relations that close the system and that ensure its compatibility without any supplementary approximation. They are derived by manipulating the equations (Eqs. 1-2) corresponding to the different faces, and by introducting auxiliary fields and auxiliary equations on each edge (N^2 per edge) and each corner (N^3 per corner). The result is a multi-dimensional solver with equations to be solved on the volume, the faces, the edges and the corners of Ω. See [1] for a time-dependent version of this solver.

As preliminary 3D finite element results, we present simulations of a spherical wave generated inside a cubic domain (Fig. 1). HABCs are prescribed on all the faces of the domain. By comparing the three cases, we see the positive effect of both the compatibility relations and the rotating branch cut on the quality of the solution. When using both strategies (Fig. 1f), the remaining error corresponds to the classical numerical dispersion caused by the mesh.

References

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