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In the case of very thin surface coatings, the coating layer contribution is often discarded in a large-scale Finite Element Analysis (FEA), especially if the coating is considered to be significantly softer than the substrate. Furthermore, special purpose structural elements like shells or membranes do not capture the in-layer deformation of soft surface coatings on hard substrates. Also considering elements with reduced dimension or high aspect ratio add to the ill-conditioning by introducing high eigenmodes in the system. Therefore, performing a large-scale analysis with a soft thin coatings is often avoided due to extensive numerical costs involved to capture the correct behaviour in the surface layer. However, not always the surface coating can be discarded, especially in contact environments.

To overcome the excessive computational cost introduced by the full discretization of the thin layer in large-scale structural contact simulations an alternative approach needs to be considered. For thin layers it is shown by Greenwood and Barber [1] that main asymptotic solutions converge to a certain asymptote. This asymptote is defined by a polynomial function by Johnson [2]. Matthewson's early work [3] show that the indentation-induced deformation of a thin elastic layer bonded to the rigid surface can be approximated by a finite power series. It is shown that for thin layers a simple order two polynomial expansion can provide a good approximation for the top surface displacement. One can see potential in using polynomial expansion based approximation for the surface layer together with FEA to provide a more general numerical framework including deformable substrates and complex geometries.

Let us propose that the layer bonded to a deformable substrate Ω can be viewed as a surface Γ described by a dimensional variable $\hat{y}$. Furthermore, the displacement field in the coating layer can be approximated by a finite polynomial expansion defined as $P_i(\hat{y})$, including the unknown displacement modes $\boldsymbol{u}_i$. In this case the displacement $\boldsymbol{u}_a(x, y)$ in the whole 2D domain can be written in a compact form as

$$\boldsymbol{u}_a(x,y) = \begin{cases} \boldsymbol{u}(x,y), & \text{in } \Omega \\ \boldsymbol{u}_0(x) + \displaystyle\sum_{i=1}^{n} P_i(\hat{y})\boldsymbol{u}_i(x), & \text{on } \Gamma \end{cases}$$

where the terms $\boldsymbol{u}_0$ symbolise the layer to substrate coupling terms, making the layer coupling to the fully discretized substrate straightforward. Thus making the method easily suitable for usage with FEA.

We take the integral over the layer dimensional variable $\hat{y}$ using a quadrature rule, whereas for linear elasticity one can even evaluate the integral analytically. The terms left have become surface integrals over Γ. Therefore, the two phase problem is changed by an interface problem, involving only domain Ω and interface Γ, that lies on the domain boundary $\partial\Omega$. As the boundary is discretized for FEM, the stiffness contribution at each boundary element can be directly added to the global FE-system. This can even be done by defining a special purpose element involving all the displacement modes.

In this research we present the methodology to consider a polynomial approximation of a thin coating behaviour in a general FE contact analysis. The main correction to standard Kuhn-Tucker conditions is the modification of the gap function to take into account the displacements on the interface. We consider a linear constraint based formulation for the interface tracking, allowing the direct use of any general purpose FE contact algorithm, like Lagrange or Penalty method. This formulation leads us to a significant computational cost reduction compared to fully discretized layer alternative without the need for special purpose contact algorithm.

However, a polynomial power series approximation results in an exponential increase of the conditioning number [4], rendering the system ill-conditioned for higher order terms. This problem is analysed by considering alternative forms of the polynomial series $P(\hat{y})$, like improved power series, Lagrange or Bernstein polynomials.

It is shown that for certain cases there is a significant cost reduction when considering polynomial based approximation of thin surface layers in contact, especially regarding soft layers on harder substrates. Making it a feasible solution to be considered in contact simulations of large-structures with thin surface coatings. The method requires no volumetric layer discretization as the thickness of the layer is a mesh independent input parameter. Therefore it can also be easily used with complex geometries.

References

[1] Greenwood J., Barber J., "Indentation of an elastic layer by a rigid cylinder," *International Journal of Solids and Structures* ***49*** *(21), 2962-2977 (2012).*

[2] Johnson K.J., *Contact mechanics*, Cambridge university press (1987).

[3] Matthewson M.J., "Axi-symmetric contact on thin compliant coatings," *Journal of the Mechanics and Physics of Solids* ***29*** *(2), 89-113 (1981).*

[4] Strang G., Fix G.J., *An analysis of the finite element method*, Prentice-hall Englewood Cliffs, NJ (1973).