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► **To cite this version:**

Benjamin Cotté. Extended source models for wind turbine noise propagation. Journal of the Acoustical Society of America, 2019, 145 (3), pp.1363-1371. 10.1121/1.5093307 . hal-02362199

HAL Id: hal-02362199

<https://hal.science/hal-02362199>

Submitted on 13 Nov 2019

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Extended source models for wind turbine noise propagation

B. Cotté^{1, a)}

Institute of Mechanical Sciences and Industrial Applications (IMSI),

ENSTA ParisTech, CNRS, CEA, EDF, Université Paris-Saclay

828 bd des Maréchaux, 91120 Palaiseau, France

(Dated: 31 January 2019)

1 Accurate prediction of wind turbine noise propagation over long distances requires
2 to model the dominant broadband aerodynamic noise sources, as well as the main
3 outdoor sound propagation effects. In this study, two methods are compared to
4 include extended aeroacoustic source models in a parabolic equation code for wind
5 turbine noise propagation in an inhomogeneous atmosphere. In the first method, an
6 initial starter is obtained for each segment of the blade using the backpropagation
7 approach. In the second method, the blade segments are viewed as moving monopole
8 sources, and only a limited number of parabolic equation simulations are needed
9 for different source heights across the rotor plane. The two methods are compared
10 to the point source approximation first in a homogeneous medium for validation
11 purposes, and then in a stratified inhomogeneous atmosphere. The results show that
12 an extended source model is necessary to calculate the sound pressure level upwind,
13 where a shadow zone is present, and to obtain the correct amplitude modulation
14 levels. Furthermore, the second method is seen to yield as accurate results as the first
15 method when a sufficient number of source heights is considered, with a computation
16 time that is much reduced.

^{a)}benjamin.cotte@ensta-paristech.fr

17 I. INTRODUCTION

18 Wind turbine noise can be perceived at distances greater than one kilometer and is char-
19 acterized by amplitude modulations at the receiver (Larsson and Öhlund, 2014; Zajamsek
20 *et al.*, 2016). As noise restrictions limit the areas where onshore wind farms can be built,
21 an accurate prediction of the far-field noise is needed in order to improve the placement of
22 the turbines at a given site, as well as to develop noise mitigation methods. This requires
23 to model the dominant broadband aerodynamic noise sources as well as the main outdoor
24 sound propagation effects that occur between the wind turbines and the receivers. The main
25 aerodynamic noise sources are generally considered to be turbulent inflow noise, correspond-
26 ing to the interaction of atmospheric turbulence with the blade leading edge, and trailing
27 edge noise, corresponding to the scattering of the turbulent boundary layer at the blade
28 trailing edge. As shown in the experimental campaign of Buck *et al.* (2016), turbulent in-
29 flow noise is generally dominant at low frequencies, typically below 300-400 Hz for a modern
30 upwind turbine, while trailing edge noise dominates at higher frequencies, as already shown
31 by Oerlemans and Schepers (2009).

32 To model aerodynamic noise sources, the state-of-the-art approach is to divide the wind
33 turbine blades into radial segments, and to sum incoherently the noise contributions from
34 each segment at the receiver locations (Oerlemans and Schepers, 2009; Zhu *et al.*, 2005).
35 To model atmospheric propagation, however, this approach is rarely used, and it is more
36 common to model the wind turbine as a point source of specified power located at the rotor
37 center (Lee *et al.*, 2016; Prospathopoulos and Voutsinas, 2007). Recently, several methods

38 have been proposed to include an extended source model in wind turbine noise propagation
39 calculations. [McBride and Burdisso \(2017\)](#) and [Heimann *et al.* \(2018\)](#) have considered
40 extended models in ray-based models. [McBride and Burdisso \(2017\)](#) have kept all the blade
41 segments used in the aeroacoustic source model in their 3D ray-tracing approach, while
42 [Heimann *et al.* \(2018\)](#) consider 24 fixed point sources distributed over the rotor disk with an
43 identical sound power level (no source model used). One of the known weaknesses of these
44 ray-based models is the treatment of diffraction, for instance in the presence of an acoustic
45 shadow zone. Other authors have proposed methods based on the parabolic equation (PE),
46 that is able to treat diffraction effects accurately. [Barlas *et al.* \(2017\)](#) have considered a
47 PE model considering only one point source per blade. This point source is located at the
48 segment location where the maximum noise level is calculated by their aerodynamic noise
49 source model for each frequency. [Cotté \(2018\)](#) has kept several segments along the blade,
50 and has used the backpropagation method to preserve the directivity of the noise sources,
51 which makes the approach very computationally demanding.

52 In this study, two methods are compared to include extended aeroacoustic source models
53 in a parabolic equation code for acoustic propagation in an inhomogeneous atmosphere. The
54 source model is based on Amiet's theory ([Roger and Moreau, 2010](#); [Tian and Cotté, 2016](#)),
55 but the methods could be applied to other source models, such as the so-called BPM semi-
56 empirical model that is widely used in wind turbine noise prediction studies ([Oerlemans and](#)
57 [Schepers, 2009](#); [Zhu *et al.*, 2005](#)). In the first method, that was recently proposed ([Cotté,](#)
58 [2018](#)), an initial starter for the PE model is obtained for each segment of the blade using the
59 backpropagation approach. In the second method, that is introduced in the present study,

60 the blade segments are viewed as moving monopole sources, and only a limited number of
 61 parabolic equation simulations are needed which strongly reduces the computation time.
 62 The two methods are compared to the point source approximation, first in a homogeneous
 63 medium for validation purposes and second in a stratified atmosphere.

64 The paper is organized as follows. In Section II, the source and propagation models are
 65 described, as well as the proposed extended source methods. Then, the models are compared
 66 in Section III, first in a homogeneous atmosphere to be validated against an analytical
 67 solution, and then in an inhomogeneous atmosphere to take into account refraction effects
 68 in different directions of propagation.

69 II. DESCRIPTION OF THE EXTENDED SOURCE MODELS

70 A. Description of the broadband noise sources using strip theory

71 It is common in the literature to calculate the noise spectrum of a full blade using strip
 72 theory (Christophe *et al.*, 2009; Rozenberg *et al.*, 2010; Sinayoko *et al.*, 2013). This theory
 73 consists in dividing the blade into M small segments or strips along the radial direction in
 74 order to take into account the variation of the blade geometry and the incident flow, as
 75 schematically shown in Fig. 1(a). Each segment is represented as an airfoil of chord c_m and
 76 span L_m , $m = 1, \dots, M$. The different segments are supposed to be uncorrelated, so that the
 77 noise contributions from all blade segments can be summed at the receiver. This assumption
 78 is one of the main limitation of strip theory at low frequencies, as discussed by Christophe
 79 *et al.* (2009).

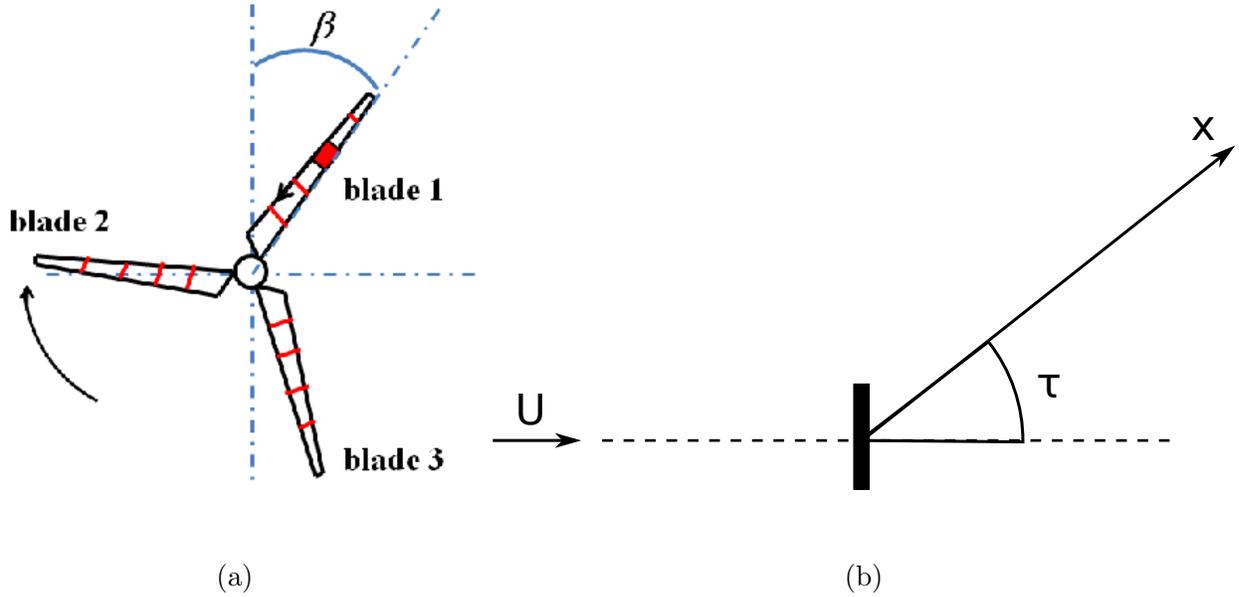


FIG. 1. Notations for (a) the rotor plane with blades divided into segments, and (b) the wind turbine propagation in a direction τ with respect to the direction of the wind U .

80 For each segment at each angular position β , the power spectral density (PSD) of the
 81 acoustic pressure p for the rotating airfoil at angular frequency ω is written (Sinayoko *et al.*,
 82 2013; Tian and Cotté, 2016):

$$S_{pp}^R(\mathbf{x}_R^T, \omega, \beta) = \frac{\omega_e}{\omega} S_{pp}^F(\mathbf{x}_R^B, \omega_e, \beta), \quad (1)$$

83 with ω_e the emission angular frequency, \mathbf{x}_R^T the receiver coordinates in the wind turbine
 84 reference system, \mathbf{x}_R^B the receiver coordinates in the blade reference system, and S_{pp}^F the
 85 PSD for an airfoil that is fixed relative to the receiver. The expression for the Doppler
 86 factor ω/ω_e is given in Sinayoko *et al.* (2013).

87 In order to calculate the PSD of acoustic pressure S_{pp}^F for an airfoil that is fixed relative
 88 to the receiver, various methods have been proposed in the literature. In this study, a model
 89 of trailing edge noise and turbulent inflow noise for wind turbines based on Amiet's theory

90 is used, that is detailed in [Tian and Cotté \(2016\)](#). The model is valid for an aspect ratio
 91 $L_m/c_m \geq 3$, $m = 1..M$, and the receiver is supposed to be in the far-field.

92 **B. Acoustic propagation model based on the parabolic approximation**

93 The acoustic propagation model considered here is a parabolic equation with fractional
 94 steps, called split-step Padé, based on higher order Padé approximants and solved with the
 95 method of [Collins \(1993\)](#). It has been shown in [Cotté \(2018\)](#) for a typical wind turbine
 96 configuration that it is more computationally effective than a classical wide-angle parabolic
 97 equation based on a Padé (1,1) approximation of the propagation operator. In this study,
 98 the effective sound speed approximation is used, which allows one to take into account the
 99 refraction effects due to the vertical wind gradients in the equation for a medium at rest:

$$c_{\text{eff}}(z) = c(z) + U(z) \cos \tau = \sqrt{\gamma_0 r T(z)} + U(z) \cos \tau, \quad (2)$$

100 with z the height above ground, γ_0 the specific-heat ratio, r the specific gas constant, $U(z)$
 101 and $T(z)$ the mean vertical profiles of wind speed and temperature, and τ the angle between
 102 the wind direction and the propagation direction from the source to the receiver noted as x ,
 103 as shown in Fig. 1(b). In order to introduce the notations needed for the extended source
 104 models described in Secs. II C and II D, the main equations of the model are briefly reminded
 105 below.

106 Using the axisymmetric approximation, the three-dimensional Helmholtz equation can
 107 be reduced to the following two-dimensional equation in the far-field:

$$\left[\frac{\partial^2}{\partial x^2} + \left(\frac{\partial}{\partial z^2} + k^2 \right) \right] q_c = 0, \quad (3)$$

108 where $q_c = p_c \sqrt{x}$ connects the q_c variable to the complex pressure p_c , and k is the acoustic
 109 wavenumber. This wavenumber can be written as $k^2 = k_0^2 n^2 = k_0^2 (1 + \epsilon)$, where $n(z) =$
 110 $c_0/c_{\text{eff}}(z)$ is the index of refraction and $k_0 = \omega/c_0$ is the value of the acoustic wavenumber
 111 at the reference sound speed c_0 . Introducing the propagation operator

$$\mathcal{Q} = \left(1 + \epsilon + \frac{1}{k_0^2} \frac{\partial}{\partial z^2} \right)^{1/2} = (1 + \mathcal{L})^{1/2}, \quad (4)$$

112 that is independent of x in range-independent media, Eq. (5) becomes (Gilbert and White,
 113 1989):

$$\left(\frac{\partial}{\partial x} + ik_0 \mathcal{Q} \right) \left(\frac{\partial}{\partial x} - ik_0 \mathcal{Q} \right) q_c = 0. \quad (5)$$

114 We can decouple Eq. (5) into two equations characterizing a wave propagating in the positive
 115 x direction, denoted as q_+ (propagating wave), and a wave propagating in the negative x
 116 direction denoted as q_- (backpropagating wave). Using the notation $\gamma = \pm 1$, one obtains
 117 from Eq. (5):

$$\left(\frac{\partial}{\partial x} - i\gamma k_0 \mathcal{Q} \right) q_\gamma = 0. \quad (6)$$

118 Introducing the variable ϕ_γ corresponding to the envelope of the pressure:

$$q_\gamma(x, z) = \phi_\gamma(x, z) \exp(i\gamma k_0 x), \quad (7)$$

119 and substituting Eq. (7) into Eq. (6), the following equation is obtained:

$$\frac{\partial \phi_\gamma}{\partial x} = i\gamma k_0 (\mathcal{Q} - 1) \phi_\gamma. \quad (8)$$

120 To solve Eq. (8), the domain is discretized using a rectangular mesh of size Δx and Δz
 121 along the x and z -axis respectively. The split-step Padé (N,N) method is used to advance
 122 the field from x to $x + \Delta x$ for $\gamma = 1$, or from x to $x - \Delta x$ for $\gamma = -1$ (Collins, 1993; Dallois

123 *et al.*, 2002). The angular validity increases with the order N of the development and depends
 124 on the mesh size Δx chosen (Dallois *et al.*, 2002). As shown in Cotté (2018), accurate results
 125 are obtained in a typical wind turbine configuration with $N = 2$ and mesh sizes $\Delta x = 2\lambda$ and
 126 $\Delta z = \lambda/10$, where λ is the acoustic wavelength. Along the vertical direction, the domain is
 127 bounded by a ground impedance condition at $z = 0$, and by an absorbing layer at the top
 128 of the domain to obtain non-reflecting boundary conditions (Salomons, 2001).

129 C. Extended source model based on the backpropagation method (Amiet-PE 130 model)

131 The first extended source model considered in this study is based on the parabolic equa-
 132 tion property to decouple forward and backward-propagating waves, as shown in Eqs. (6)
 133 and (8). It was proposed by Cotté (2018) and is called Amiet-PE model. The basics of the
 134 method are summarized in this section.

135 For each segment m , each angular position β of the blade and each angular frequency
 136 ω , a parabolic equation calculation is performed for which an initial condition at $x = 0$ is
 137 needed. This initial condition is obtained numerically using the backpropagation method,
 138 whose principle is illustrated in Fig. 2(a). It consists first in back-propagating a known
 139 pressure field, that is noted “initial solution” in Fig. 2(a), at $x = x_{is}$ to $x = 0$, taking
 140 $\gamma = -1$ in the equations. Then, in a second step, the starter at $x = 0$ is forward-propagated
 141 to the desired distance using the “classical” parabolic equation with $\gamma = +1$.

142 In the backpropagation method, the initial solution at $x = x_{is}$ is obtained for heights
 143 $z_{is,p} = p\Delta z, p = 0..P$, from the expression (1) for the PSD of acoustic pressure of a rotating

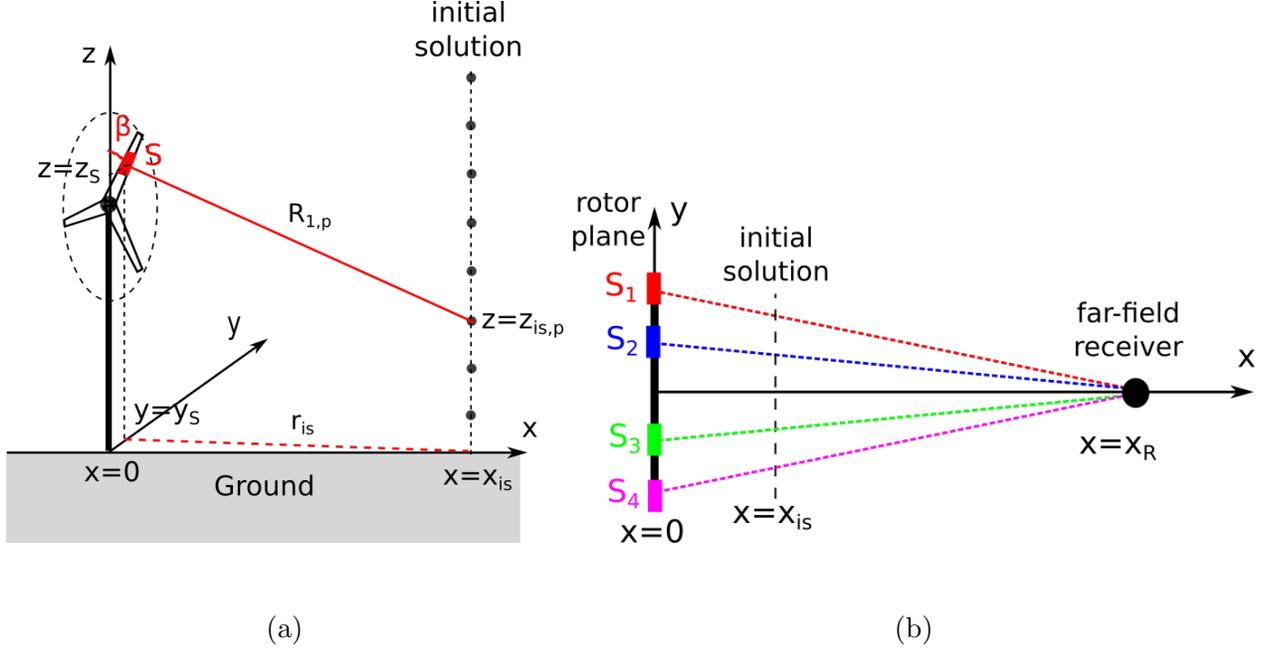


FIG. 2. Schematics for (a) the backpropagation method used to obtain the starter at $x = 0$ from the initial solution at $x = x_{is}$ for one blade segment S , and (b) the different propagation planes between 4 blade segments noted S_1 , S_2 , S_3 and S_4 and the far-field receiver at $x = x_R$ (top view).

Color online.

144 blade. The initial solution thus includes the source directivity as viewed by this vertical
 145 line of receivers in this specific direction. In the presence of ground, the initial solution is
 146 written:

$$q_c(z_{is,p}) = \sqrt{S_{pp}^R(\mathbf{x}_R^T, \omega, \beta)} \sqrt{x_S} e^{ik_0 R_{1,p}} \times \left(1 + Q \frac{R_{1,p}}{R_{2,p}} e^{ik_0(R_{2,p} - R_{1,p})} \right), \quad (9)$$

147 where $R_{1,p} = \sqrt{r_{is}^2 + (z_S - z_{is,p})^2}$ and $R_{2,p} = \sqrt{r_{is}^2 + (z_S + z_{is,p})^2}$ are respectively the dis-
 148 tance between the segment at $(0, y_S, z_S)$ or the image segment at $(0, y_S, -z_S)$ and the p^{th}
 149 initial starter point, with $r_{is} = \sqrt{x_{is}^2 + (y_S - y_{is})^2}$, and Q is the spherical wave reflection
 150 coefficient.

151 During the backpropagation calculation, the atmosphere is supposed homogeneous and
 152 the ground is taken as rigid ($Q = 1$). Then, the starter at $x = 0$ can be propagated using
 153 any ground impedance and any sound speed profile. Note also that each calculation is per-
 154 formed in a slightly different plane that crosses the far-field receiver at $x = x_R$, as shown in
 155 Fig. 2(b). This method is therefore strictly exact only at this distance. For $x \neq x_R$, the total
 156 acoustic pressure is obtained by summing contributions with different y values. Since the
 157 radius of the rotor (typically 50 m) is generally small compared to the propagation distances
 158 considered, the method remains valid over a wide range of distances. The computational
 159 cost of this method is quite high, since MN_β PE calculations per frequency and per prop-
 160 agation direction need to be performed, where N_β is the number of angular positions used
 161 to discretize the rotor plane.

162 **D. Extended source model based on moving monopoles (MM model)**

163 The second extended source model considered in this study represents each segment of
 164 the blade as a monopole rotating at angular velocity $\dot{\beta}$. It is called the moving monopoles
 165 (MM) model. Compared to the first method, it does not rely on the parabolic approximation
 166 and can be applied to any propagation model.

167 In the MM model, the sound pressure level (SPL) at the receiver is calculated for a
 168 segment m at angular position β using the point source approximation (Salomons, 2001):

$$\begin{aligned} \text{SPL}(\omega, \beta) = & \text{SWL}(\omega, \beta) - 10 \log_{10}(4\pi R_1^2) \\ & + \Delta L(\omega, \beta) - \alpha(\omega)R_1, \end{aligned} \tag{10}$$

169 where $\text{SWL}(\omega, \beta)$ is the angle-dependent sound power level (SWL), $R_1 = \sqrt{x^2 + y_S^2 + (z_S - z)^2}$
 170 is the distance between the segment at $(0, y_S, z_S)$ and the receiver at $(x, 0, z)$, ΔL is the
 171 sound pressure relative to the free field, and α is the absorption coefficient in dB/m.

172 The angle-dependent SWL can be obtained from the free-field SPL calculated using
 173 Amiet's model. Assuming free-field conditions ($\Delta L = 0$) and no absorption in the medium,
 174 Eq. (10) becomes:

$$\begin{aligned} \text{SWL}(\omega, \beta) &= \text{SPL}_{FF}(\omega, \beta) + 10 \log_{10}(4\pi R_1^2) \\ &= 10 \log_{10} \left(\frac{S_{pp}^R(\mathbf{x}_R^T, \omega, \beta)}{p_{\text{ref}}^2} \right) + 10 \log_{10}(4\pi R_1^2), \end{aligned} \quad (11)$$

175 with SPL_{FF} the free-field SPL and $p_{\text{ref}} = 20 \mu\text{Pa}$ the reference pressure. From Eqs. (10) and
 176 (11), the following equation for $\text{SPL}(\omega, \beta)$ is obtained:

$$\begin{aligned} \text{SPL}(\omega, \beta) &= 10 \log_{10} \left(\frac{S_{pp}^R(\mathbf{x}_R^T, \omega, \beta)}{p_{\text{ref}}^2} \right) \\ &\quad + \Delta L(\omega, \beta) - \alpha(\omega)R_1. \end{aligned} \quad (12)$$

177 In Eq. (12), the main unknown is the relative sound pressure level $\Delta L(\omega, \beta)$. For the
 178 propagation over a finite impedance ground in a homogeneous atmosphere at rest, it can be
 179 calculated analytically (Salomons, 2001):

$$\Delta L = 10 \log_{10} \left| 1 + Q \frac{R_1}{R_2} e^{ik_0(R_2 - R_1)} \right|^2, \quad (13)$$

180 with $R_2 = \sqrt{x^2 + y_S^2 + (z_S + z)^2}$ the distance between the image-source and the receiver.

181 In order to include refraction effects, $\Delta L(\omega, \beta)$ can be calculated using the parabolic ap-
 182 proximation method described in Sec. II B. The initial starter corresponding to the monopole
 183 source is calculated numerically using the backpropagation method in order to preserve the
 184 angular validity of the split-step Padé (2,2) method (Galindo, 1996). In order to limit the

185 number of PE calculations to perform, a set of N_h source heights distributed along the rotor
 186 plane are considered:

$$H_n = H_{\min} + n\Delta H, \quad n = 0, \dots, N_h - 1, \quad (14)$$

187 with ΔH the height step given by:

$$\Delta H = \frac{H_{\max} - H_{\min}}{N_h - 1}, \quad (15)$$

188 where H_{\min} and H_{\max} are respectively the minimum and maximum heights to consider. The
 189 relative sound pressure level $\Delta L(\omega, \beta)$ in Eq. (12) is then obtained using a nearest-neighbor
 190 interpolation. As an example, the monopole sources are represented for the three blades in
 191 Fig. 3 with $M = 6$ segments per blade. Using $N_h = 5$ source heights in the MM model, the
 192 sources are shifted to a fictive position determined by the nearest-neighbor interpolation,
 193 as shown by the arrows in Fig. 3. The maximum difference between the fictive and exact
 194 source heights is thus $\Delta H/2$. Note that these fictive positions are only used to calculate
 195 $\Delta L(\omega, \beta)$ in Eq. (10), since the variables $\text{SWL}(\omega, \beta)$ and R_1 are calculated from the exact
 196 source positions.

197 As a result, there are N_h PE calculations to perform per frequency and per propagation
 198 direction in the MM model. The computational cost of MM model is thus reduced compared
 199 to the Amiet-PE model since $N_h < MN_\beta$ in practice. On the other hand, the MM model
 200 does not consider the source directivity in the vertical direction.

201 Note finally that the point source approximation is a special case of the MM model,
 202 where only one PE calculation is performed for a source located at the hub height. It is still
 203 possible in this case to obtain the evolution of the SPL with respect to the angular position

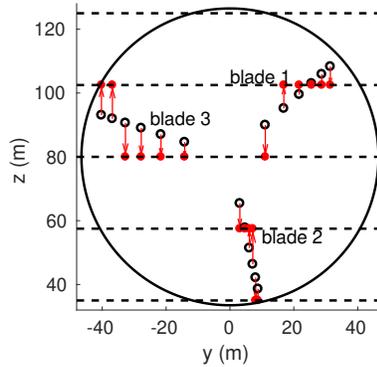


FIG. 3. Exact (\circ) and fictive (\bullet) positions of the monopole sources distributed along each blade in the MM model at $\beta = 48^\circ$ with $M = 6$ segments and $N_h = 5$ source heights represented as horizontal dashed lines ($\Delta H = 22.5$ m).

204 β using Eq. (12), which will be necessary to calculate the amplitude modulation in Sec. III.
 205 This means that the point source approximation is only used to account for propagation
 206 effects in the present study.

207 III. RESULTS USING BOTH EXTENDED SOURCE MODELS

208 A. Configurations studied

209 In this study, the same 2.3 MW wind turbine as in Tian and Cotté (2016) and Cotté (2018)
 210 is considered, with a diameter of 93 m, a hub height of 80 m and three blades of length 45 m.
 211 As justified in Tian and Cotté (2016), each blade is decomposed into $M = 8$ segments to
 212 respect the constraint on the aspect ratio $L_m/c_m \geq 3$, $m = 1, \dots, M$ mentioned in Sec. II A.
 213 The rotation of the blade is divided into $N_\beta = 30$ angular positions (resolution of 12°). The

214 wind velocity at at the hub height $z = 80$ m is assumed to be 8 m/s, and the angular velocity
 215 of the rotor is 13 rpm.

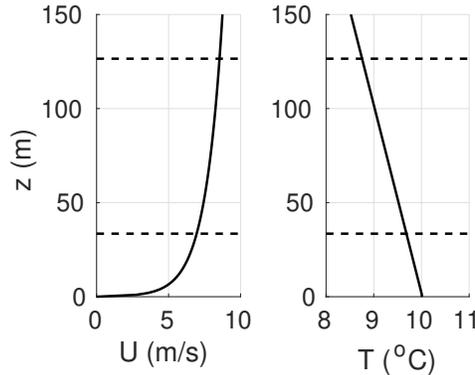


FIG. 4. Vertical profiles of wind speed $U(z)$ and temperature $T(z)$ in a neutral atmosphere. The minimum and maximum rotor heights are represented as horizontal dashed lines.

Two test cases are considered to evaluate the accuracy of the Amiet-PE and MM models. In the first case, only trailing edge noise is included, and the wind speed profile is assumed to be constant in the source model (no wind shear). The propagation conditions are assumed to be homogeneous ($c(z) = c_0$), with a finite impedance ground. The absence of refraction effects makes it possible to compare the results of the coupled model with the analytical solution in a homogeneous atmosphere based on Equations (12) and (13). In the second test-case, both trailing edge and turbulent inflow noise sources are considered, and the atmosphere is supposed to be neutral. Using the Monin-Obukhov similarity theory, this means that the vertical profiles of the mean wind speed $U(z)$ and of the temperature $T(z)$

are given by (Salomons, 2001):

$$U(z) = \frac{u_*}{\kappa} \ln \left(\frac{z}{z_0} \right), \quad (16)$$

$$T(z) = T_0 + \alpha_0 z, \quad (17)$$

216 where $u_* = 0.49$ m/s is the friction velocity, $z_0 = 0.1$ m is the surface roughness length,
 217 $T_0 = 10^\circ\text{C}$ is the ground temperature, $\alpha_0 = -0.01$ K/m is the dry adiabatic lapse rate, and
 218 $\kappa = 0.41$ is the von Kármán constant. The value of the friction velocity is chosen so that
 219 $U(z = 80 \text{ m}) = 8$ m/s. The vertical profiles of $U(z)$ and $T(z)$ are plotted in Fig. 4. In both
 220 test-cases, the scattering effect of turbulence is not included in the model, which means that
 221 the SPL might be underestimated when a shadow zone is present (Cotté, 2018, Section 4.4).

222 The propagation domain has a size of 1200 m along x and 300 m along z . PE calculations
 223 are performed for 49 frequencies in order to predict the third octave band spectra between
 224 100 Hz and 2000 Hz (Cotté, 2018). The ground impedance is calculated with a two-parameter
 225 variable porosity model, which is physically admissible and yields a better agreement with
 226 measurements than commonly used one-parameter models (e.g. Delany-Bazley or Miki), as
 227 shown by Dragna *et al.* (2015). The effective resistivity is $\sigma_e = 50$ kNs/m⁴ and the rate of
 228 change of the porosity is $\alpha_e = 100$ m⁻¹, that are typical values for a natural soil (Dragna
 229 *et al.*, 2015, Table III).

230 In the Amiet-PE model, the initial starter is computed at a distance $x_{is} = 100$ m, and
 231 the far-field receiver is placed at $x_R = 1000$ m; see Fig. 2(b). In the MM model, the number
 232 of source heights N_h varies between 3 and 19, which corresponds to a height step ΔH
 233 decreasing from 45 m down to 5 m, considering $H_{\min} = 35$ m and $H_{\max} = 125$ m. To give an

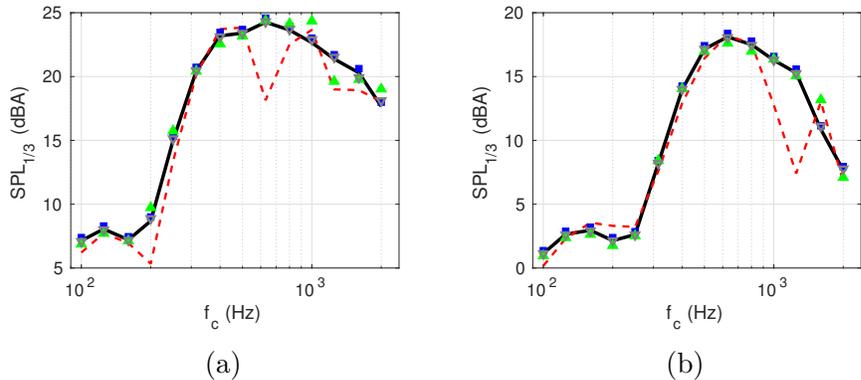


FIG. 9. Third octave band spectrum of the SPL downwind ($\tau = 0^\circ$) at $z = 2$ m and (a) $x = 500$ m or (b) $x = 1000$ m: analytical solution (\blackrightarrow), point source approximation ($-\cdot-$), Amiet-PE (\blacksquare), MM with 3 heights (\blacktriangle) or 7 heights (\blacktriangledown). Color online.

234 order of magnitude of the computation time, a set of PE calculations for the 49 frequencies
 235 takes approximately 8 minutes to run on one core of a PC equipped with an Intel Xeon
 236 X5650 processor at 2.66 GHz. For each direction τ , the computation time of the MM model
 237 is thus between approximately 24 minutes with $N_h = 3$ and 2h30' with $N_h = 19$, and the
 238 computation time of the Amiet-PE model is greater than 30 hours.

239 B. Validation in a homogeneous atmosphere

240 First, the third octave band spectra of SPL averaged over one rotation are plotted in
 241 Fig. 5 for a receiver at a height of 2 m and at a distance of 500 m or 1000 m downwind
 242 ($\tau = 0^\circ$). The results with the point source approximation, the Amiet-PE model, and
 243 the MM model with three and seven source heights are compared to the analytical solution.
 244 Using the point source approximation, there are fluctuations due to ground interference dips
 245 that are much reduced using an extended source model. These fluctuations are still visible
 246 in the MM model with 3 source heights. Excellent agreement is found with the analytical
 247 solution using either the Amiet-PE model or the MM model with 7 source heights.

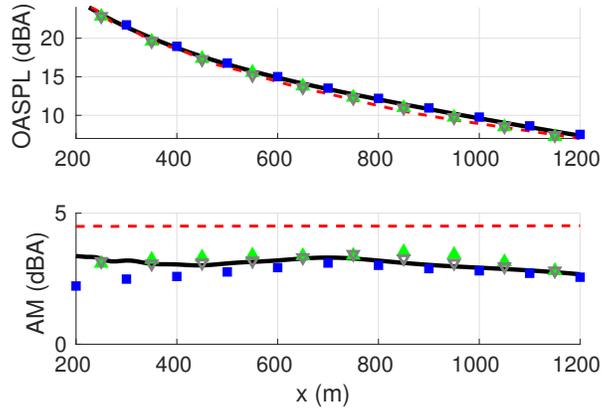


FIG. 6. OASPL and AM versus x at $z = 2$ m crosswind ($\tau = 90^\circ$): analytical solution (—), point source approximation (---), Amiet-PE (■), MM with 3 heights (▲) or 7 heights (▼). Color online.

248 Then, the overall sound pressure level (OASPL) averaged over one rotation and the
 249 amplitude modulation (AM) are plotted as a function of x in Fig. 6 for a receiver at a height
 250 of 2 m crosswind ($\tau = 90^\circ$). The amplitude modulation is defined as the difference between
 251 the maximum and the minimum of the OASPL over one rotation. On the one hand, all
 252 the OASPL predictions are within 1 dB(A) from the analytical calculation, even with the
 253 point source approximation. On the other hand, AM is seen to be much more sensitive to
 254 the source model used. As explained in Sec. IID, the point source approximation is only
 255 used to account for propagation effects in the present study, so it is theoretically possible to
 256 calculate AM using this source model, although the predicted value does not agree with the
 257 analytical solution. Using the Amiet-PE model, the AM predictions are accurate only for
 258 distances larger than 500 m approximately, which can be attributed to the fact that receivers
 259 at short ranges are far from the point at $x_R = 1000$ m where all the propagation planes cross,

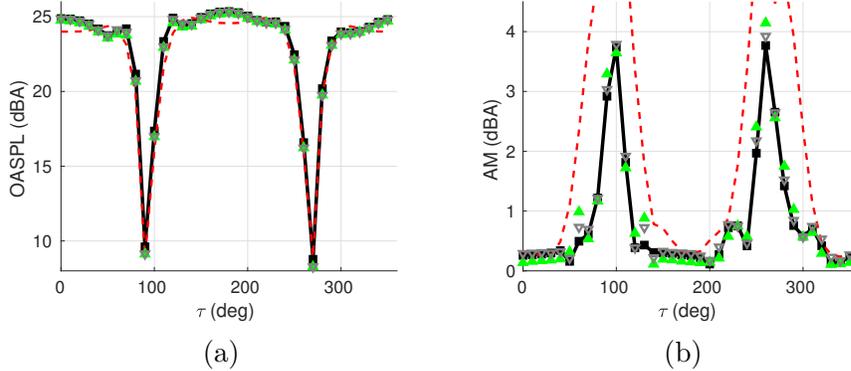


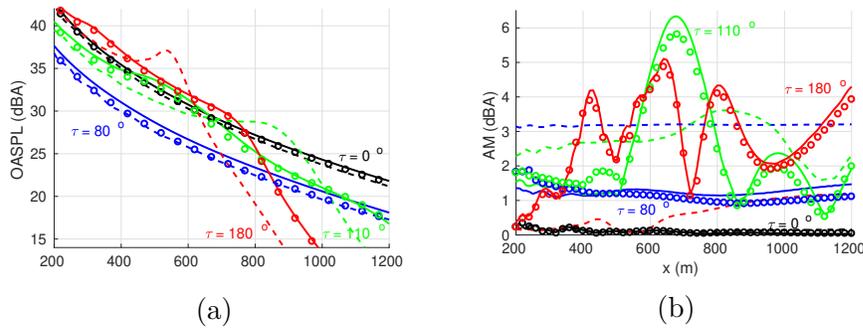
FIG. 7. Directivity of (a) OASPL and (b) AM at $x = 1000$ m and $z = 2$ m: analytical solution (—), point source approximation (-- --), MM with 3 heights (\blacktriangle) or 7 heights (∇). Color online.

260 as schematically shown in Fig. 2(b). The MM model yields very accurate AM values when
 261 at least 7 source heights are considered.

262 Finally, the directivities of OASPL and AM at a distance 1000 m and a height of 2 m
 263 are plotted in Fig. 7, using an angular step $\Delta\tau = 10^\circ$. The Amiet-PE calculations are
 264 not shown because it would be too computationally expensive with such a small value of
 265 $\Delta\tau$. In the OASPL directivity plot, some differences compared to the analytical solution
 266 are obtained with the point source approximation, that remain smaller than 1 dB(A) except
 267 close to the interference dips, while very accurate results are obtained using the MM model
 268 with 7 heights. The AM predictions using the point source approximation are completely off,
 269 while the ones obtained with the MM model are quite accurate, especially with 7 heights.
 270 The MM model predictions with 10 and 19 heights are not shown as they are almost
 271 identical to the predictions with 7 heights.

272 C. Results in a neutrally stratified atmosphere

273 In a neutral atmosphere, wind turbine noise propagation is completely different downwind
 274 and upwind, due to the presence of a shadow zone in the latter case. This is clearly seen



with respect to x at $z = 2$ m in a neutral atmosphere at $\tau = 0^\circ$ (downwind), $\tau = 80^\circ$, $\tau = 110^\circ$, and $\tau = 180^\circ$ (upwind): Amiet-PE (—), point source approximation (---), MM with 19 heights (—○). Color online.

275 in the top plot of Fig. 8(a), where the evolution of OASPL with distance is plotted for
 276 propagation directions downwind, crosswind and upwind. Instead of considering the exact
 277 crosswind direction ($\tau = 90^\circ$), where the OASPL is very low, as seen in Fig. 7, two directions
 278 close to crosswind have been chosen: $\tau = 80^\circ$ and $\tau = 110^\circ$. For distances greater than
 279 approximately 800 m, the models predict a rapid decrease of the OASPL upwind, due to
 280 the shadow zone effect. This decrease is also seen for $\tau = 110^\circ$ although it is more gentle.
 281 Note that the shadow zone effect might be less pronounced in a real atmosphere, since the
 282 scattering effect due to turbulence has not been included in the present model, as discussed
 283 in Cotté (2018). In the directions $\tau = 0^\circ$ and $\tau = 80^\circ$, almost identical OASPL predictions
 284 are obtained using the Amiet-PE model, the MM model with 19 source heights and the
 285 point source approximation. In the directions $\tau = 110^\circ$ and $\tau = 180^\circ$, on the other hand,
 286 the point source approximation yields large errors at long distances. In the upwind direction,
 287 for instance, the shadow zone starts approximately 200 m earlier compared to the extended
 288 source model calculations.

289 To quantify the error made using various methods, let us define the maximum difference
 290 over a quantity $A(x)$ as:

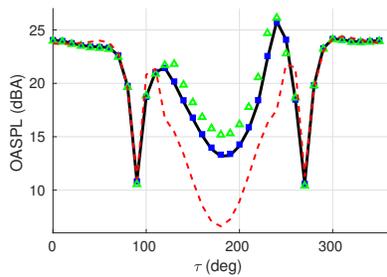
$$\text{MaxDiff}(A) = \max_{500 \text{ m} \leq x \leq 1200 \text{ m}} |A(x) - A_{\text{ref}}(x)|, \quad (18)$$

291 where A_{ref} is a reference calculation. In Table I, the maximum difference $\text{MaxDiff}(\text{OASPL})$
 292 is given using various models considering the Amiet-PE model as a reference. Note that
 293 the maximum difference is only calculated between 500 m and 1200 m in Eq. (18) as the
 294 validity of the Amiet-PE model is questionable at short ranges, as discussed in Sec. III B,
 295 and because the dwellings are generally located at least 500 m from the closest wind turbine.
 296 Table I shows that the point source approximation yields maximum differences greater than
 297 1.0 dB in the crosswind and upwind directions. The MM model yields accurate results in all
 298 directions if at least 10 source heights are considered.

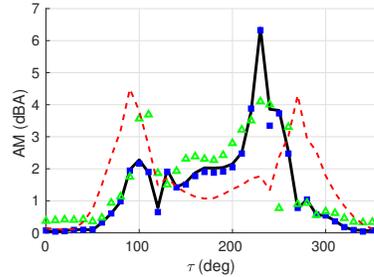
299 The evolution of AM with distance is plotted in Fig. 8(b) for the same four directions.
 300 The AM remains smaller than 0.2 dB(A) downwind, with similar results for all models. In
 301 the other directions, the AM is much higher, and is not well predicted using the point source
 302 approximation. The high values of the AM in upward-refracting conditions ($\tau = 110^\circ$ and
 303 $\tau = 180^\circ$) for distances greater than 400 m are attributed to the fact that the receiver will
 304 enter and leave the illuminated region during the blade rotation when it is close to the limit
 305 of the shadow zone, as shown in Barlas *et al.* (2017) and Cotté (2018). Since the shadow zone
 306 starts at a shorter range for higher frequency, the highest AM is encountered at different
 307 frequencies depending on the receiver positions (Cotté, 2018). In Table II, the maximum
 308 difference $\text{MaxDiff}(\text{AM})$ is given using the Amiet-PE model as a reference. It can be seen

TABLE I. Maximum difference MaxDiff(OASPL) with respect to the Amiet-PE model using the point source (PS) approximation and the MM model with 3 heights (MM3), 7 heights (MM7), 10 heights (MM10) and 19 heights (MM19) for different angles of propagation τ . Boldface values correspond to differences strictly greater than 1.0 dB.

τ	PS	MM3	MM7	MM10	MM19
0°	0.6 dB	0.4 dB	0.3 dB	0.3 dB	0.3 dB
80°	1.1 dB	1.0 dB	0.9 dB	0.8 dB	0.9 dB
110°	5.6 dB	3.6 dB	1.2 dB	0.8 dB	0.8 dB
180°	11.0 dB	2.6 dB	0.4 dB	0.4 dB	0.3 dB



(a)



(b)

FIG. 9. Directivity of (a) OASPL and (b) AM at $x = 1000$ m and $z = 2$ m in a neutral atmosphere: MM with 19 heights (\blackrightarrow), MM with 10 heights (\blacksquare), MM with 3 heights (\blacktriangle), and point source approximation ($-\cdot-$). Color online.

309 that at least 10 source heights are needed in the MM model in order to obtain a maximum
 310 difference smaller than 1.1 dB in all directions.

311 To confirm that the MM model predictions converge with increasing source heights in
 312 all propagation directions, the directivities of OASPL and AM are plotted in Fig. 9 at a
 313 distance of 1000 m and a height of 2 m. Using the MM model with 19 source heights as
 314 the reference calculation, the difference between the OASPL predictions are observed in the

TABLE II. Maximum difference MaxDiff(AM) with respect to the Amiet-PE model using the point source (PS) approximation and the MM model with 3 heights (MM3), 7 heights (MM7), 10 heights (MM10) and 19 heights (MM19) for different angles of propagation τ . Boldface values correspond to differences strictly greater than 1.0 dB.

τ	PS	MM3	MM7	MM10	MM19
0°	0.1 dB	0.4 dB	0.1 dB	0.0 dB	0.0 dB
80°	2.1 dB	0.2 dB	0.3 dB	0.3 dB	0.4 dB
110°	3.4 dB	4.0 dB	1.3 dB	1.1 dB	0.5 dB
180°	4.7 dB	3.8 dB	1.4 dB	1.0 dB	0.4 dB

315 upwind directions ($100^\circ \leq \tau \leq 260^\circ$), with differences up to 8.3 dB(A) for the point source
 316 approximation, 2.4 dB(A) for the MM model with 3 heights, and only 0.3 dB(A) for the
 317 MM model with 10 heights. The same behavior is observed in the AM directivities, with
 318 differences up to 4.6 dB(A) for the point source approximation, 3.1 dB(A) for the MM model
 319 with 3 heights, and only 0.5 dB(A) for the MM model with 10 heights.

320 The movie Mm. 1 shows how the OASPL and AM horizontal directivities vary for dis-
 321 tances between 200 m and 1200 m every 10 m. The relative contributions of trailing edge
 322 noise and turbulent inflow noise are also plotted, as can be seen in Fig. 10 where two snap-
 323 shots of the movie corresponding to $x = 300$ m and $x = 1000$ m are shown. Up to 300 m
 324 approximately, refraction effects are small and the OASPL horizontal directivity keeps a
 325 dipole shape, as classically measured and predicted at short range (Buck *et al.*, 2016; Oer-

326 [lemans and Schepers, 2009](#); [Zhu *et al.*, 2005](#)). At longer ranges, the OASPL directivity
 327 changes to an asymmetric shape, with small levels upwind, as seen for instance in [Barlas](#)
 328 [et al. \(2017\)](#) and [McBride and Burdisso \(2017\)](#). The AM directivity shows some peaks in
 329 various upwind directions depending on the propagation distance for $x > 400$ m. This can be
 330 attributed to the influence of the acoustic shadow zone, as explained previously. Note that
 331 significant AM values have also been reported in the downwind directions in other studies.
 332 For instance, [Barlas *et al.* \(2017\)](#) have obtained high AM values due to the effect of the wind
 333 turbine wake on acoustic propagation. This effect is not included in the present calculations.

334 Mm. 1. Directivity of OASPL and AM calculated with the MM model using $N_h = 10$ source
 335 heights with respect to distances between $x = 200$ m and $x = 1200$ m at $z = 2$ m in a
 336 neutral atmosphere. The trailing edge noise (noted TEN) is shown in red, the turbulent
 337 inflow noise (noted TIN) is shown in blue, and the total prediction (noted Total) is shown
 338 in black. The wind is blowing from the left. File of type “avi” (8.4 MB)

339 It is also interesting to note in movie Mm. 1 that the OASPL directivities for trailing
 340 edge noise and turbulent inflow noise become quite different at large distances. This can be
 341 observed in the third octave band spectra of [Fig. 11](#) at a distance of 1000 m in the downwind
 342 direction ($\tau = 0^\circ$), in the direction where turbulent inflow noise is dominant ($\tau = 120^\circ$), and
 343 in the direction where trailing edge noise is dominant ($\tau = 240^\circ$). At $\tau = 0^\circ$, the balance
 344 between the two noise generation mechanisms is similar to the one seen in the sound power
 345 level spectra, with trailing edge noise being dominant at high frequencies ($f > 250$ Hz) and
 346 turbulent inflow noise being dominant at low frequencies ($f < 250$ Hz). On the other hand,

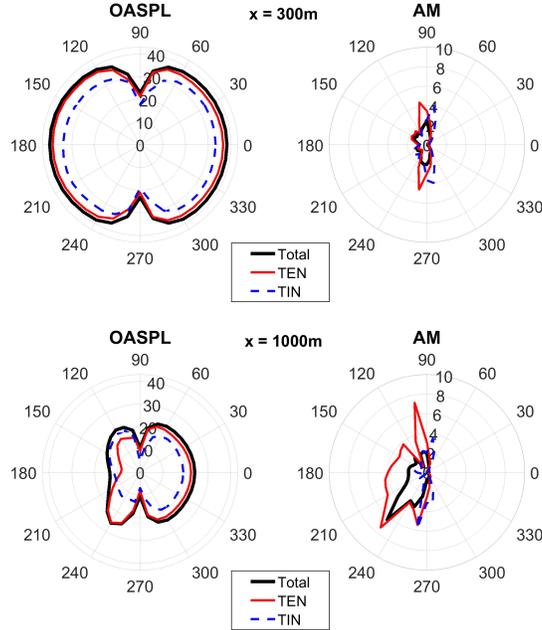


FIG. 10. Directivity of OASPL and AM calculated with the MM model using $N_h = 10$ source heights at $z = 2$ m and $x = 300$ m (top) or $x = 1000$ m (bottom) in a neutral atmosphere. The thick solid line corresponds to the total prediction (noted Total), the thin solid line to the trailing edge noise (noted TEN), and the thin dashed line to turbulent inflow noise (noted TIN). The wind is blowing from the left.

347 turbulent inflow noise becomes dominant for most frequencies at $\tau = 120^\circ$, while trailing edge
 348 noise becomes dominant for most frequencies at $\tau = 240^\circ$.

349 IV. CONCLUSION

350 In this study, two methods have been tested to include extended aeroacoustic source
 351 models in a parabolic equation code for wind turbine noise propagation in an inhomogeneous
 352 atmosphere. These two methods have been compared to the point source approximation that

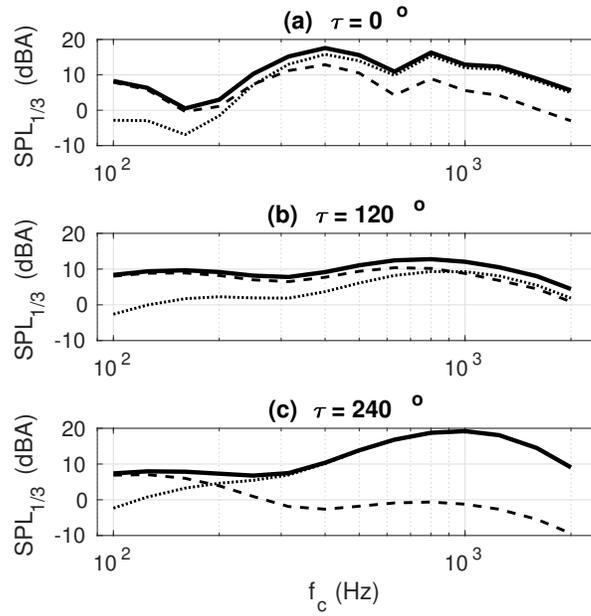


FIG. 11. Third octave band spectrum of the SPL in a neutral atmosphere at $x = 1000$ m and $z = 2$ m calculated with the MM model with 10 source heights at (a) $\tau = 0^\circ$, (b) $\tau = 120^\circ$, and (c) $\tau = 240^\circ$: total prediction (solid lines), trailing edge noise only (dotted lines), and turbulent inflow noise only (dashed lines).

353 is classically used in wind turbine noise propagation studies. The source model is based on
 354 Amiet's theory, and the parabolic equation code uses a split-step Padé approximant. In the
 355 first method, called Amiet-PE, an initial starter is obtained for each segment of the blade
 356 using the backpropagation approach. This method enables one to accurately model the
 357 directivity of the noise sources but is very computationally intensive. In the second method,
 358 the blade segments are viewed as moving monopole sources (MM model), and only a limited
 359 number of parabolic equation simulations are needed depending on the number of source
 360 heights considered to discretize the rotor plane.

361 The various models are first validated using an analytical reference solution in a homo-
362 geneous medium. The Amiet-PE model and the MM model with at least 7 source heights
363 ($\Delta H \leq 15$ m) are in excellent agreement with the reference solution, in terms of spec-
364 tra, OASPL and AM. The point source approximation is relatively accurate to predict the
365 OASPL, but it is unable to predict the AM, and tends to exaggerate the ground interference
366 dips in the spectra, even at large distances from the source.

367 The models are then compared in a neutrally stratified atmosphere, characterized by a
368 logarithmic velocity profile. The most challenging propagation conditions are encountered
369 upwind, where an acoustic shadow zone appears for propagation distances greater than
370 approximately 400 m. The point source approximation fails to calculate the correct OASPL
371 in these directions, because it predicts a shadow zone that starts too close to the wind
372 turbine. In order to correctly capture the AM behavior upwind, the Amiet-PE model and
373 the MM model with at least 10 source heights ($\Delta H \leq 10$ m) are shown to yield accurate
374 results. The MM model is much more computationally effective than the Amiet-PE model,
375 with a ratio $MN_\beta/N_h \approx 24$ between the two models with $N_h = 10$ source heights.

376 The MM model proposed in this article could be used in the future to study the effect
377 of strong wind speed gradients, that is potentially the source of amplitude modulation at
378 night (van den Berg, 2008; Zajamsek *et al.*, 2016), the influence of the wind turbine wake on
379 propagation (Barlas *et al.*, 2017), or the combined influence of topography and meteorology,
380 using for instance the rotated PE approach described in Lihoreau *et al.* (2006).

381 **ACKNOWLEDGMENTS**

382 The author would like to thank Benoit Gauvreau and David Ecoti re from the Environ-
383 mental Acoustics Unit (UMRAE), and Tommy Rigall from IMSIA for their useful comments
384 on the manuscript.

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