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AN ANALYTICAL APPROACH BASED ON TAILORED GREEN'S FUNCTIONS FOR FLOW NOISE PREDICTION AT LOW MACH NUMBER

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The presence of rigid boundary surfaces in a turbulent flow can result in the enhancement of the radiated acoustic field. Two contributions can be distinguished: a direct contribution of quadrupolar nature and a diffracted contribution produced by the scattering of the direct field by the geometry. In order to predict the global acoustic radiation, the method of Green's functions is a particularly convenient choice. Provided an appropriate Green function, which can be found analytically for simple geometries or numerically for arbitrary shapes, and appropriate statistics for the incompressible turbulent flow, an estimation of both contributions can be made. In this study, we derive an analytical method based on Green's functions satisfying the rigid boundary condition over the boundary surfaces which can be applied to arbitrary geometries.

The fundamental equation governing the generation of sound in the presence of rigid boundaries in the frequency domain is [1]:

$$p(\mathbf{x},\omega) = \int_V \frac{\partial^2 G_T(\mathbf{x},\mathbf{y},\omega)}{\partial y_i \partial y_j} T_{ij}(\mathbf{y},\omega) dV(\mathbf{y}), \quad (1)$$

where V is the turbulence volume surrounding a solid boundary S, T_{ij} the Lighthill stress tensor, $\mathbf{x}$ the observer location, $\mathbf{y}$ the source location and where $G_T(\mathbf{x},\mathbf{y},\omega)$ is the tailored Green function, satisfying the rigid boundary condition on S, defined by:

$$\begin{aligned} &(\Delta + k_0^2) G_T(\mathbf{x},\mathbf{y},\omega) = \delta(\mathbf{x}-\mathbf{y}) \\ &\frac{\partial G_T}{\partial y_i} n_i = 0, \quad \forall \mathbf{y} \in S. \end{aligned} \quad (2)$$

For a turbulent flow, the source term T_{ij} can be considered as a random function and is therefore not square-integrable. The computation of (1) is then not straightforward and a solution is to compute the power spectral density (PSD) which is defined by:

$$S_{pp}(\mathbf{x},\omega) = \langle p(\mathbf{x},\omega) p^*(\mathbf{x},\omega) \rangle, \quad (3)$$

where * denotes the complex conjuguate and $\langle\rangle$ the ensemble average. In trailing edge noise study, it is usual to consider an incompressible turbulent boundary layer over a semi-infinite plate such as described in Fig. 1. Also, considering the classical Reynold's decomposition $u_i = U_i + u_i'$, if we assume that only the transverse mean shear $\partial U_1/\partial y_2$ is non zero and that the velocity components can be related by an anisotropy factor so that $u_i' = f_{ij} u_j'$, which will be taken equal to 1 is this study, (1) can be writen:

$$\begin{aligned} p(\mathbf{x},\omega) = \int_V & 2\rho_0 U_1(y_2) u_2'(\mathbf{y},\omega) \\ & \times \frac{\partial}{\partial y_1} \left(\sum_{i=1}^{3} \frac{\partial G_T}{\partial y_i} \right) dV(\mathbf{y}). \end{aligned} \quad (4)$$

The exact Green function, tailored to the semi-infinite plate is given for an observer in the far field ($|\mathbf{x}| \to \infty$) by [2]:

$$\begin{aligned} G_T(\mathbf{x},\mathbf{y},\omega) \approx & -\frac{1}{4\pi} \\ & \times \sum_{m=0}^{\infty} a_m \cos\left(\nu_m(\theta_x+\pi)\right) \cos\left(\nu_m(\theta_y+\pi)\right) \\ & \times J_{\nu_m}\left(k_0 r_y \frac{r_x}{|\mathbf{x}-y_3\mathbf{e_3}|}\right) \frac{e^{ik_0|\mathbf{x}-y_3\mathbf{e_3}|}}{|\mathbf{x}-y_3\mathbf{e_3}|} e^{-i\pi m/4}, \end{aligned} \quad (5)$$

where $\nu_m = m/2$, $r_x = \sqrt{x_1^2+x_2^2}$, $r_y = \sqrt{y_1^2+y_2^2}$, $a_0 = 1$ and $a_m = 2$ for $m \neq 0$. Because the only assumption is that the observer is placed in far field, both direct and diffracted fields are included in this formulation.

Replacing (4) in (3) yield to an expression of the PSD as a function of $\phi_{22}(\mathbf{y},\mathbf{z},\omega) = \langle u_2'(\mathbf{y},\omega) u_2'^*(\mathbf{z},\omega) \rangle$, the cross spectral density (CSD) of the vertical component of the turbulent fluctuation velocity. From the Blake-TNO wall pressure spectrum model, widely used in applications of the Chase's diffraction theory [3, 4], we derived a semi-analytical model scaled by a XFoil simulation. Considering the turbulence as isotropic and as homogeneous in the directions parallel to the plane the CSD can be written under the following separable formulation:

$$\phi_{22}(\mathbf{y},\mathbf{z},\omega) = \overline{u_2'^2}(y_2) \phi_{22}^{(1)}(r_1,\omega) L_2(y_2) \phi_{22}^{(3)}(r_3,\omega) \quad (6)$$

where $r_i = y_i - z_i$ represents the correlation separation and $\overline{u_2'^2}(y_2)$ is the variance of the turbulence fluctuations. The one dimensional spectrum in the streamwise direction is derived from the von Kármán model [5] under the frozen

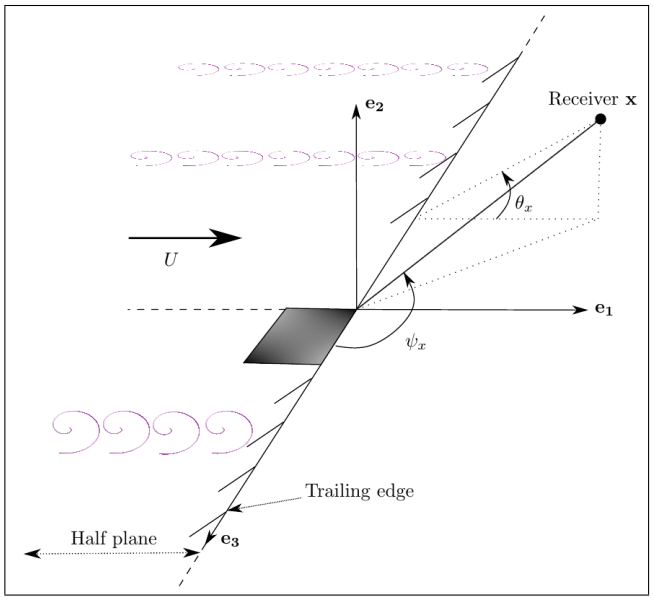


Figure 1. Typical configuration in trailing edge noise studies

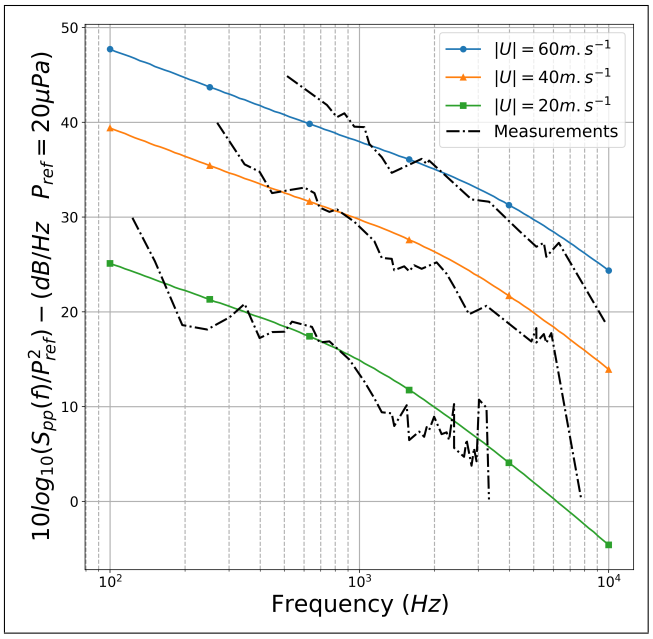


Figure 2. Sound pressure level (dB=Hz) for a receiver above the trailing edge at $r_x = 1.22m$ at mid-span ($\theta_x = \psi_x = \pi = 2$) calculated using the far field half plane Green function and measured by Stalnov et al. [4]

turbulence assumption and yields:

$$\phi_{22}^{(1)}(r_1,\omega) = \frac{4}{15\sqrt{\pi}k_e U_c}\frac{\Gamma(11/6)}{\Gamma(7/3)} \times \frac{1+8/3(k_1/k_e)^2}{(1+(k_1/k_e)^2)^{11/6}} e^{-(i+\gamma)\omega r_1/U_c}, \quad (7)$$

where k_e is the energy bearing wavenumber, U_c the mean convection velocity and γ an empirical parameter of order 0.2 to 0.3. In the transverse direction a Gaussian correlation function is assumed:

$$\phi_{22}^{(3)}(r_3,\omega) = e^{-(r_3/\lambda_3)^2}, \quad (8)$$

where $\lambda_3 = 1.4U_c/\omega$ is a transverse correlation scale given by Corcos [6]. Finally, if the correlation length scales of the turbulence are small compared to the spatial extend of the boundary layer then the CSD can be expressed as a function of integral length scales only:

$$\phi_{22}(\mathbf{y},\mathbf{z},\omega) = \overline{u_2'^2}(y_2)L_1(\omega)L_2(y_2)L_3(\omega)\delta(\mathbf{y}-\mathbf{z}), \quad (9)$$

where L_i is an integral length scale defined by,

$$L_i(\omega) = \int_{-\infty}^{\infty} \phi_{22}^{(i)}(r_i,\omega)dr_i. \quad (10)$$

In order to validate the noise spectrum prediction against experimental results, we consider a NACA 0012 airfoil in different configurations for which trailing edge noise is the dominant contribution. Fig. 2 shows that the predicted spectra at $U = 20m.s^{-1}, 40m.s^{-1}, 60m.s^{-1}$ of a profile of chord $0.2m$ and span $0.45m$ at zero angle of attack for an observer placed at $1.22m$ above the trailing edge are in good acccordance with the measured spectra. Power law results deduced from classical dimensional analysis are recovered: the direct acoustic field, produced by eddies far from the edge increases in proportion to the fourth power of the Mach number and the diffracted field directly in proportion to the Mach number.

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