Robust Motion Planning of the Powered Descent of a Space Vehicle

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Abstract: The motion planning of powered descent problems has often been treated in the deterministic optimal control framework, which provides efficient theoretical and numerical tools. However, future applications require robustness, usually obtained by introducing stochastic components in the dynamics to model uncertainties. After stating the robust motion planning problem, this paper proposes a deterministic approximation which avoids the computational difficulties of stochastic optimal control. The approach consists of guiding the mean while reducing the covariance, whose dynamics are approximated thanks to statistical linearization. Moreover, since feedback control is necessary to control covariance, two manners to cope with actuator saturations with a stochastic control are provided.

Keywords: Trajectory and Path Planning, Aerospace, Robust Control, Control of constrained systems, Output feedback control

1. INTRODUCTION

By the renewed interest these last years for Mars and Moon exploration, with new missions requiring high landing precision, and the growing use of reusable launchers, it has become necessary to devise efficient vertical landing strategies, as argued in Blackmore (2016). A challenge in vertical landing is its last phase, the powered descent, beginning a few kilometers above the landing site with a limited amount a fuel remaining. It is usually achieved through feedback control, that allows to reach the target precisely using the available measurements by tracking a reference trajectory computed thanks to motion planning.

The motion planning has frequently been treated in an optimal control framework, seeking for the control succeeding to steer the vehicle to a target while minimizing a cost function. The control structure of the optimal control during powered descent has been studied numerous times in the litterature, from Meditch (1964) and its study of the fuel-saving one dimensional lunar landing problem. Recently, Leparoux et al. (2022b) found that the general structure of optimal solutions of powered descent problems considering control and state constraints is the Max-Min-Max form, which corresponds to a control of the engine throttle that switches twice between its maximum and minimum levels. This form of solution is well suited for rocket engines because it does not require thrust modulation, which is hard to achieve. However, since the control is always saturated, it leaves no room to adapt to perturbations or in case of errors on the initial conditions, possibly high when landing on planets other than Earth according to Braun and Manning (2007).

Those uncertainties may be considered by introducing a white noise in the dynamics and modeling the vehicle state as a random variable. Then, the robustness of the motion planning is ensured if the solution allows to limit the dispersions around the mean trajectory. This leads to a stochastic formulation of the motion planning problem, which is not desirable since few theoretical results exist and numerical methods in the stochastic optimal control framework are computationally heavy. In the literature, several references propose covariance control approaches through feedback control for linear dynamics. For instance, Chen et al. (2015) uses a LQG regulator to steer the state of a linear system to a specified terminal Gaussian distribution. Moreover, Ridderhof and Tsiotras (2019) presents an algorithm for powered descent that considers separately the mean steering which has the role to bring the vehicle to the target optimally, and the covariance steering to attain prescribed final dispersions. When considering non linear dynamics, Berret and Jean (2020) proposes to use statistical linearization to approximate the first two moments of the stochastic state, and then to approximate the motion planning solution by the one of an open-loop stochastic optimal control problem.

Thus, it appears that leveraging feedback control to achieve robust motion planning through covariance control is promising. However, this requires to take into account actuator saturations. For powered descent problem, several ideas have been proposed to this end. For instance, in Shen et al. (2010), which treats the robust powered descent problem by reducing the solution sensitivity rather than covariance, the feedback gain is multiplied by a factor that vanishes when the reference control is close to the saturation. In Ridderhof and Tsiotras (2019), the control saturations are handled by convexifying control constraints formulated as chance constraints to apply them on the mean control. Moreover, Ridderhof and Tsiotras (2021) extends this approach by calculating conservative margins that allow to respect control bounds above a given probability threshold.

This paper proposes a robust motion planning approach for powered descent problems. This approach considers a realistic space vehicle model, with non linear dynamics, actuator saturations and state constraints. Uncertainties are modeled by the introduction of stochastic components in the dynamics and the initial conditions. The problem consists of steering the mean positions and velocities to a target, while minimizing a cost depending on the mean state, the control and the covariance. The dynamics of the mean and the covariance are approximated thanks to statistical linearization. In addition, saturation of the actuators is modeled and managed in two different ways. Specifically, one of the proposed solutions consists of including a smooth approximation of the saturation function directly in the dynamics.

This paper is organized as follows. First, we present the dynamics and the constraints of the landing problem considered. Then, we detail how we formulate a robust motion planning problem thanks to statistical linearization. In Section 4 we present how to include the actuator saturations and the state constraints into the problem formulation, and finally Section 5 provides some numerical results.

2. FRAMEWORK

Let us study the motion planning of a space vehicle during the powered descent phase. It is treated here as an optimal control problem, and in two dimensions for the sake of simplicity. The aim is to compute a reference trajectory and a reference control that allow the space vehicle to reach a predefined target S_f while minimizing fuel consumption.

2.1 Deterministic landing problem

First, let us describe the unperturbed dynamics of the space vehicle, when assuming that all parameters are known and that the vehicle is only affected by its weight and by the engine thrust. The state is denoted $x = (r, v, m) \in \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}$ where r is the position, v the velocity, m the mass of the vehicle and u the control. Its unperturbed dynamics are expressed by $\dot{x} = f(x, u)$ in an inertial frame (e_x, e_z), such that

$$\begin{cases} \dot{r} &= v, \\ \dot{v} &= \dfrac{T}{m}u - g_0, \\ \dot{m} &= -q\|u\|, \end{cases} \tag{1}$$

where u is the control, q the maximal mass flow rate of the engine, $T > 0$ the maximal thrust and $g_0 = (0, g_a)$ with g_a the gravitational acceleration. The initial mass of the vehicle is denoted m_0 and its empty mass m_e. Technical limitations and security requirements are modeled by state and control constraints, as explained in further details in Leparoux et al. (2022a). They should be satisfied for all $t \in [0, t_f)$ and are expressed by:

$$\begin{cases} m(t) > m_e \ ; & (2) \\ 0 \le u_{min} \le \|u(t)\| \le u_{max}; & (3) \\ \langle e_z, u\rangle \ge \|u\| \sin(\theta_{min}), \text{ with } \theta_{min} \in [0, \dfrac{\pi}{2}); & (4) \\ h(r) = r_z - \tan(\gamma)|r_x| \ge 0, \text{ with } \gamma \in [0, \dfrac{\pi}{2}). & (5) \end{cases}$$

According to Leparoux et al. (2022a)[Theorem 1.1 and Proposition 4.3], for generic initial conditions, the optimal control u that steers (r, v) following (1) to zero under the constraints (2), (3), (4), (5) while maximizing the final mass has a Max-Min-Max form, i.e. there exist t_1 and t_2 with $0 \le t_1 \le t_2 \le t_f$ such that

$$\|u(t)\| = \begin{cases} u_{max} \text{ if } t \in [0, t_1) \cup (t_2, t_f], \\ u_{min} \text{ if } t \in [t_1, t_2]. \end{cases} \tag{6}$$

2.2 Modeling of uncertainties

The motion planning problem stated above allows to compute the less expensive way to attain a target. However, in the presence of perturbations and uncertainties, constraints might be violated, which could cause the failure of the mission. In order to take into account the uncertainties when computing the solution, we model them by introducing stochastic components in the problem. In the case of the powered descent problem, several uncertainties might act on the dynamics:

- uncertainties on the initial state, due to measurement errors, and drifts on position and velocity estimates.
- perturbations of the dynamics, such as aerodynamic forces not modeled in (1). This is modeled by a white noise on the dynamics, such that

$$dx_t = f(x_t, u(t))dt + g(x_t)dW_t, \tag{7}$$

  where x_t is a stochastic state variable, $g(x_t) = \frac{\sigma(t)}{m_t}$ is a diffusion term and dW_t a monovariate Wiener process. However, we neglect the perturbations on the mass equation, by defining the standard deviation vector $\sigma \in \mathbb{R}^5$ with the last component equal to zero.

2.3 Robust motion planning

For now, we do not consider the state constraints (2) and (5). When considering a stochastic system, motion planning consists of imposing terminal constraints on the trajectory, typically imposing the mean to reach the target. Moreover, for the mean to be representative of the global behavior of the system, the dispersion of the sample trajectories must be small along the trajectories and at the final time. To this end, we add in the cost function penalizations on the integral of the covariance P_x along the trajectory and on the final covariance $P_x(t_f)$. Let us denote by $\mathcal{U} = \{u | u_{min} \le \|u\| \le u_{max}$ and $\langle e_z, u\rangle \ge \|u\| \sin(\theta_{min})\}$ the admissible control set. Then a first robust motion planning problem can be stated in a stochastic framework as follows

Problem 1. (Stochastic robust motion planning).

$$\min_{u,t_f} \quad \mathbb{E}[-m_{t_f}] + \mathrm{tr}(Q_f P_x(t_f)) + \int_0^{t_f} \mathrm{tr}(QP_x(t))dt$$

under the constraints

$$\begin{cases} x_t \text{ follows (7)}, \\ u(t) \in \mathcal{U} \ \forall t \in [0, t_f), \\ \mathbb{E}[x_{t_f}] \in \mathcal{S}_f. \end{cases}$$

3. DETERMINISTIC PROBLEM APPROXIMATION

A stochastic optimal control problem such as Problem 1 is difficult to solve because few theoretical and numerical methods are available and they are computationally expensive. In this section, we propose a deterministic approximation of Problem 1.

3.1 Statistical linearization

As explained in Section 2.3, the quantities of major interest for motion planning are the first two moments of x_t. Thus we prefer working on an approximation of Problem 1 using as state variables the mean and the covariance, denoted respectively by m_x and P_x. We approximate m_x and P_x using a statistical linearization method, as solutions $(\hat{x}, P)$ of the following differential system

$$\begin{cases} \dot{\hat{x}} &= f(\hat{x}, u), \\ \dot{P} &= D_x f(\hat{x}, u)P + PD_x f(\hat{x}, u)^\top + g(\hat{x})g(\hat{x})^\top. \end{cases} \tag{8}$$

Statistical linearization methods are used for a long time, especially in the fields of mechanics, and are well known. See for instance Berret and Jean (2020) for justifications of their use in motion planning. Thus, the control solution of Problem 1 is approximated by the control solution of the following problem

Problem 2. (Deterministic robust motion planning).

$$\min_{u,t_f} \quad -\hat{m}(t_f) + \mathrm{tr}(Q_f P(t_f)) + \int_0^{t_f} \mathrm{tr}(QP(t))dt \tag{9}$$

under the constraints

$$\begin{cases} (\hat{x}, P)(\cdot) \text{ follows (8)}, \\ u(t) \in \mathcal{U} \ \forall t \in [0, t_f), \\ \hat{x}(t_f) \in \mathcal{S}_f. \end{cases}$$

However, the viewpoint of Problems 1 and 2, and of any optimal control problem, is consistent only if there is accessibility. Indeed, if the covariance is not accessible, then there exists no solution substantially reducing it or even preventing it from increasing. Accessibility of statistically linearized systems has been studied in Bonalli et al. (2022), where a sufficient accessibility condition is clarified. We checked this condition for the launcher model described in this paper and it appears that it is not verified when considering (1) as dynamics. The calculation of the accessibility condition for the launcher example is too long to be presented in this paper, but it will appear in future work.

Remark 3. Since the condition given in the reference is only sufficient, its calculation on the launcher dynamics does not prove that the linearized launcher is not accessible in open loop. However, in light of numerical results such as simulations presented in the last section of this paper, it seems clear that it is difficult to keep the covariance small when considering controls only function of time.

3.2 Feedback for covariance control

Considering the open-loop dynamics (8) is not satisfying to solve Problems 1 and 2, as explained in Section 3.1. Since position and velocity measurements are available during powered descent, we modify the launcher dynamics by the one obtained when considering u as a function of position and velocity. However, the mass is not considered in the feedback, because it is not observable and we assume that it is not measured. Let us note first that u represents the normalized thrust of an engine, whose force and direction are controlled by different actuators. Therefore, we adopt a notation of u, which separates its norm u_ρ, and its direction u_θ, such that

$$u = u_\rho \begin{pmatrix} \cos(u_\theta) \\ \sin(u_\theta) \end{pmatrix} \tag{10}$$

where $u_\rho = \|u\|$ and $u_\theta \in [-\pi, \pi)$. Then, we assume that there are linear feedbacks on the norm and the direction, such that

$$u_{FB}(x,t) = (\rho(t) + K_n(t)\xi(t)) \begin{pmatrix} \cos(\theta(t) + K_d(t)\xi(t)) \\ \sin(\theta(t) + K_d(t)\xi(t)) \end{pmatrix} \tag{11}$$

where ρ and θ are deterministic references, K_n and $K_d \in \mathbb{R}^4$ feedback gains, and $\xi = (r, v)$ is a reduced state vector. Finally, we will seek to solve Problem 2 where the dynamics (7) that we approximate by statistical linearization (8) has for unperturbed dynamics

$$f_{FB}(x, u(t)) = f(x, u_{FB}(x,t)). \tag{12}$$

Remark 4. There are other advantages in considering feedback controls in motion planning, particularly when seeking robustness. Indeed, motion planning is used for providing a reference to a tracking controller, generally achieved using feedback control. By taking into account the feedback in motion planning, we obtain reference feedback gains which can be used to initialize the calculations of real feedback gains. Moreover, saturations of the actuators can be planned or avoided, as detailed further in the next section. The main drawback of considering feedback controls is their cost, since they require sensors or observers to acquire measurements, and processing to be operable.

However, a difficulty arises when assuming a control as (11) in Problem 2, because we cannot consider a stochastic control constraint such as $u_{BF}(x) \in \mathcal{U}$. The next Section will present how to deal with that control constraint.

4. CONSIDERATION OF ACTUATOR SATURATIONS AND STATE CONSTRAINTS

In this section, we detail how to include in Problem 2 the control and state constraints presented in Section 2.1 when assuming (12) as unperturbed dynamics. First, we present actuator saturations as the actual modeling of control constraints. Then we detail two approaches to handle these saturations in the motion planning problem and finally we include state constraints.

4.1 Saturated dynamics modeling

The launcher model given in Section 2.1 is suited in a motion planning purpose. Actually, control constraints (3) and (4) represent actuator saturations which are part of the dynamics. Let us define the following saturation function

$$\mathrm{sat}_a^b(x) = \begin{cases} a \text{ if } x < a, \\ b \text{ if } x > b, \\ x \text{ otherwise.} \end{cases} \tag{13}$$

Then, using the notation of the control introduced by (10), a more accurate description of the vehicle unperturbed dynamics is

$$f_{sat}(x,u) = f\left(x, \mathrm{sat}_{u_{min}}^{u_{max}}(u_\rho)\begin{pmatrix}\cos(\mathrm{sat}_{\theta_{min}}^{\pi-\theta_{min}}(u_\theta)) \\ \sin(\mathrm{sat}_{\theta_{min}}^{\pi-\theta_{min}}(u_\theta))\end{pmatrix}\right). \tag{14}$$

This amounts to a system of controlled dynamics when controls are unsaturated and uncontrolled dynamics otherwise. In motion planning, we only consider the controlled dynamics, and control constraints have the role of avoiding the saturated state. Therefore, in the presence of uncertainties, the control constraints should force the solution to stay away from saturations with a confidence margin. The next two subsections propose two ways of doing it when considering the feedback dynamics (12).

4.2 Control saturations as chance constraints

A common method to consider control saturations in the presence of feedback is to formulate chance constraints. Using them, the control solution should stay away from the saturations with a margin depending on the estimation of the level of uncertainties and a success probability threshold to be fixed. Let us formulate the norm and direction constraints considered in Section 2.1. First, consider the norm constraint (3) such that for all $t \in [0, t_f)$

$$\Pr(u_{min} \le u_\rho \text{ and } u_\rho \le u_{max}) > p_n \ . \tag{15}$$

Then, considering the direction constraint (4), we impose with a threshold p_d that for all $t \in [0, t_f)$

$$\Pr\left(\sin(u_\theta) \ge \sin(\theta_{min})\right) > p_d \ . \tag{16}$$

Chance constraints are well suited for statistically linearized system, since, under a Gaussian distribution hypothesis, they can be reformulated into deterministic constraints on the mean and the covariance, thanks to the following common result that we recall.

Let us consider a chance constraint such as

$$\Pr[a^\top x \le c] \ge p, \tag{17}$$

where $x \in \mathbb{R}^n$ is a stochastic variable Gaussian distributed of mean m_x and covariance P_x, $a \in \mathbb{R}^n$, c a constant and p a probability threshold. Then, (17) is equivalent to

$$a^\top m_x + \Psi^{-1}(p)\sqrt{a^\top P_x a} \le c, \tag{18}$$

where Ψ^{-1} is the inverse cumulative distribution function of the normal distribution. Therefore, (15) is equivalent to

$$\begin{aligned} & u_{max} - (\rho(t) + K_n(t)\hat{\xi}(t)) \ge \\ & \qquad \Psi^{-1}(p_n)\sqrt{(K_n(t)P_{rv}(t)K_n(t)^\top)}\ , \\ & \qquad \text{and} \\ & (\rho(t) + K_n(t)\hat{\xi}(t)) - u_{min} \ge \\ & \qquad \Psi^{-1}(p_n)\sqrt{(K_n(t)P_{rv}(t)K_n(t)^\top)}\ ; \end{aligned} \tag{19}$$

and (16) is equivalent to

$$\begin{aligned} & (\pi - \theta_{min}) - (\theta(t) + K_d(t)\hat{\xi}(t)) \ge \\ & \qquad \Psi^{-1}(p_d)\sqrt{(K_d(t)P_{rv}(t)K_d(t)^\top)}\ , \\ & \qquad \text{and} \\ & (\theta(t) + K_d(t)\hat{\xi}(t)) - \theta_{min} \ge \\ & \qquad \Psi^{-1}(p_d)\sqrt{(K_d(t)P_{rv}(t)K_d(t)^\top)}\ ; \end{aligned} \tag{20}$$

where $P_{rv} \in \mathcal{M}_4(\mathbb{R})$ is a matrix composed of the first four lines and columns of P. The drawback of this model is that the constraints may be numerically heavy to handle, because (19) and (20) constrain both the mean and the covariance.

4.3 Smooth approximation of the saturations

We propose an other approach that amounts to consider (14) as dynamics with (11) as control. This requires to approximate the saturated dynamics by smooth functions in order to make them tractable. Let us recall first that (13) can be explicited as

$$\mathrm{sat}_a^b(x) = \frac{b+a}{2} + \frac{|x-a| - |x-b|}{2}. \tag{21}$$

Therefore (13) can be approximated smoothly by replacing absolute values in (21) following

$$|x| \sim \sqrt{x^2 + \epsilon^2}, \tag{22}$$

the approximation being all the more accurate as the parameter $\epsilon > 0$ is close to zero.

4.4 State constraints

The accurate way to include (2) and (5) would be to consider an hybrid dynamics. Indeed, if (2) is not satisfied, this amounts to have (1) as dynamics with a null control since there is no fuel remaining. Assuming that the role of (5) is to avoid a crash of the vehicle, including it in the dynamics amounts to take zero as dynamics when it is not satisfied. As for control constraints, we formulate the motion planning problem in order to avoid these unlikable dynamics by constraining the state to verify (2) and (5) with a chosen level of confidence. To do so when considering hazards, we impose constraints on the mean with conservative deterministic margin. Considering the mass constraint (2), this leads to

$$\hat{m}(t) > (1+\eta)m_e \ \forall t \in [0, t_f), \tag{23}$$

where η is a margin greater than the order of magnitude of the uncertainties on mass measurements. Considering the glide-slope constraint (5), this leads to

$$\hat{r}_z - \tan(\gamma)|\hat{r}_x| \ge 0. \tag{24}$$

The robustness of the satisfaction of the glide-slope-constraint is ensured by taking γ sufficiently high.

Remark 5. Chance constraint formulations could be used to include state constraints too. However, it would require to change the definition of the glide-slope constraint, by taking a constraint cone of vertex at lower altitude than the target, so that it does not conflict with the terminal constraint. Concerning the mass constraint, a probability formulation is also possible, but we prefer to avoid adding a constraint on the mass variance since there is no feedback on the mass.

5. NUMERICAL RESULTS

Now, let us present some examples of motion planning of the powered descent problem, illustrating several formulations presented in this paper. Calculations, based on a direct method, are performed using CasADi (Andersson et al. (2019)) with python language and the IPOPT solver. For time discretization of the considered optimal control problems, a grid of 150 nodes is used. The vehicle parameters are $T = 16573N$, $u_{min} = 0.3$, $u_{max} = 0.8$, $q = 8.4294kg/s$ and $m_e = 1350kg$. The parameter $g_a = 3.71m/s^2$ corresponds to the Earth gravitational constant. The aim is to steer the vehicle to null final position and velocity, and the final time t_f is free. The initial mean state is given by $x_0 = (2000m, 1500m, 100m/s, -75m/s, 1905kg)$. The initial covariance matrix is set in order to model measurement uncertainties of the initial state, assumed to be Gaussian, such that $P^0 = \text{diag}(200m^2, 200m^2, 10(m/s)^2, 10(m/s)^2, 361kg^2)$. Along the trajectory, the dynamics is subject to perturbations of dispersion $\sigma(0) = (0, 0, 100N, 10N, 0)$. The pointing constraint angle is $\theta_{min} = 25°$ and the glide slope angle $\gamma = 5°$. The dynamics of the launcher is such that the mass covariance changes little along the trajectory. Therefore, we fix the mass margin to a conservative value $\eta = 3.7\%$ such that ηm_e is superior to two times the initial mass dispersion. To generate the results, the following approach was applied: first the solution of Problem 2 was calculated with $Q = \text{diag}(200, 200, 100, 100)$ and $Q_f = \text{diag}(1000, 2000, 100, 100)$. Then the dynamics (8) was simulated using the computed reference control and feedback gains and the resulted trajectories and controls are plotted. The deterministic fuel-saving optimal solution for $Q = Q_f = 0$ is plotted in red on the first set of plots, in order to serve as a comparison.

The first set of simulations (Fig. 1) shows the solution of Problem 2 in blue with a deterministic control u, i.e. without feedback. The above plot shows examples of random trajectories and their mean, and the below plots the optimal control found as solution. The control norm obtained is almost always saturated, and the final time $t_f = 68.2s$ is smaller than in the deterministic case ($t_f = 78.9s$). This seems to be a consequence of the lack of accessibility of the covariance, which forces the solution to choose another way to limit its growth, by reducing the duration of the trajectory. Therefore, it is a not surprising that covariance stays high, with a final state standard deviation of $(233.9m, 259.5m, 3.2m/s, 5.0m/s)$. The second sets of plots (Fig. 2) shows the solution with a stochastic control following (11) and actuator saturations modeled as chance constraints ($p_n = p_d = 0.99$). The cost function is now the sum of (9) and a penalization of the feedback gains and their derivative:

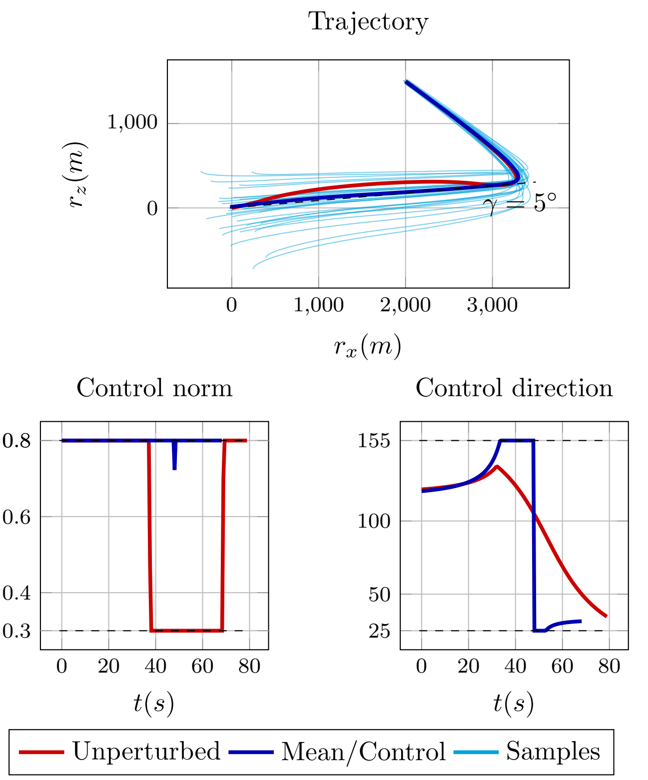

Fig. 1. Simulations of random trajectories with an open loop control

$$\min_{\rho,\theta,K_n,K_d,t_f} \quad -\hat{m}(t_f) + \text{tr}(Q_f P(t_f)) + \int_0^{t_f} \text{tr}(QP) + 0.5(\|K_n\|^2 + \|K_d\|^2 + \|\dot{K}_n\|^2 + \|\dot{K}_d\|^2)dt. \tag{25}$$

The dispersion is controlled all along the trajectory, and the final state standard deviation is $(12.2m, 9.7m, 1.4m/s, 1.0m/s)$, and $t_f = 78.7s$. Finally, the third set of plots (Fig. 3) shows the solution of the problem with control expressed by (11) and a smooth modeling of actuator saturations, with $\epsilon = 0.02$ for the norm saturation and $\epsilon = 0.0002$ for the direction. The cost is still expressed by (25). The final state standard deviation is $(12.8m, 9.9m, 1.2m/s, 0.9m/s)$ and the final time $t_f = 80.4s$.

Remark that the two stochastic controls are alike the deterministic solution, regarding the shape and the duration, except that there are margins between the extreme values of the control and the norm and direction bounds. By setting to greater values the probability parameters p_n and p_d in the second approach and the ϵ parameters in the third approach, we would obtain solutions with greater margins that would ensure better robustness with respect to control saturations. However, this might cause convergence issues, in particular for the second approach. Here, the third solution takes bigger margins from the saturations, while attaining similar final dispersions. Moreover, since the ϵ parameter in the third approach influences the smoothness of the solution, it can be adjusted so that the reference control fits realistic engine dynamics. Finally, the saturation dynamics approximation approach is adaptable outside the context of statistical linearization since it only requies mean estimations for the margins calculations, while in the

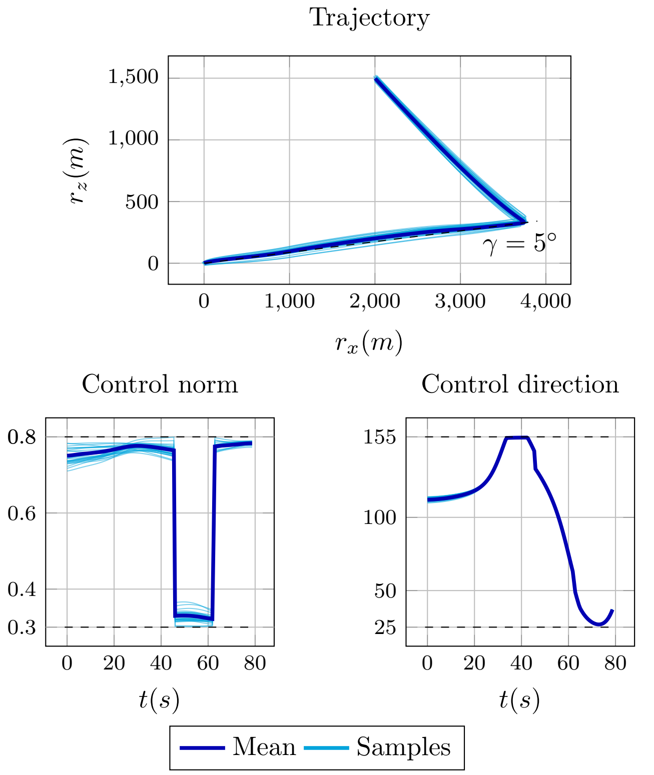

Fig. 2. Simulations of random trajectories minimizing the covariance, with control chance constraints

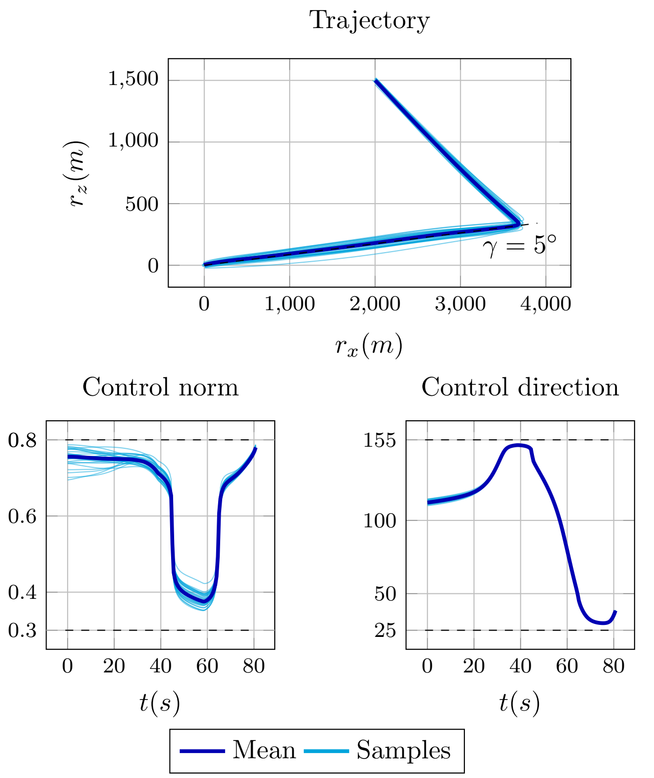

Fig. 3. Simulations of random trajectories minimizing the covariance, when actuator saturations are included in the dynamics

chance constraints approach the margins depend on both mean and covariance estimations.

6. CONCLUSION

In this paper, we have presented a robust deterministic approximation of the motion planning problem of powered descent under uncertainties. It is based on an approximation of the mean and the covariance of the stochastic state using statistical linearization. The vehicle dynamics are expressed when considering a feedback form of control, following the observations that the covariance should be limited in order to ensure the robustness of the solution, and that it requires that the control depends on the state. Then, two ways are proposed to handle actuator saturations, the first using chance constraints, and the second modeling the saturations with smooth approximations. State constraints are also considered as constraints on the mean with conservative margins. Finally, numerical results obtained by Monte-Carlo simulations show the consistency of the solutions obtained by the proposed approach.

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